

Addendum 297: The Selection Rule Is Not in the Amplitude

A fourth excluded mechanism class for OI-287-1, and a sharpening

TOE Corpus

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Skill: gap-closure v1.1 — **Outcome:** Case D (mechanism class excluded; OI-287-1 sharpened, not closed) — **Addresses:** OI-287-1 — **Verifier:** `verify_P297.py` — all PASS

Abstract

The selection rule (A296, OI-287-1) states that charged-lepton family n draws the degree- n coefficient of the density ρ . Three mechanism classes for this index matching are already excluded: the universal first-order coupling (A292), the time-average dressing at the wrong order in κ (A295), and the bare diagonal Taylor probe (A296). Each perturbed the observation operator from outside. This addendum tests the remaining inside reading: that the eigenfunctions ψ_n of the unperturbed operator already carry degree- n character, so that index matching is a property of the bare spectrum rather than an inserted coupling. The reading is excluded. The position-moment matrix elements $\langle \psi_n | x^k | \psi_n \rangle$ carry no diagonal preference: they are monotone in both indices and indistinguishable from the structureless box, and their normalized diagonal ratios $(1, 0.74, 0.59)$ run opposite to the target $(1, 1.5, 8)$. The probe leaves one positive structural fact. Under the Dirichlet condition derived in A281, every eigenfunction has leading power one at the center; the families therefore cannot be distinguished by amplitude at the origin, only by node count, the n -th level having $n - 1$ interior zeros. OI-287-1's surviving form is correspondingly narrower: whatever couples level n to degree n acts through the nodal or derivative structure of the spectrum, not through position-moment overlap.

1 The Test and Its Result

Proposition 1.1 (No index matching in the position moments). *Let $H = -d^2/dx^2 + V(x)$ with $V = [\rho'(x)]^2/(2\Omega^2) + E_{\text{self}} x^2(1-x)^2$ and $\rho'(x) = 48\pi^3 x^2 + 6\pi^2 x + 2\pi$, Dirichlet on $[0, 1]$, as fixed in P03 and used by `verify_P296.py`. Let ψ_1, ψ_2, ψ_3 be its lowest three eigenfunctions, identified with the families e, μ, τ . Form $A_{nk} = \langle \psi_n | x^k | \psi_n \rangle$ for $k = 1, 2, 3$. After standardizing each column across the three levels, no row attains its maximum on the diagonal: the arg-max pattern is (k_3, k_1, k_3) , not (k_1, k_2, k_3) . The same construction on the structureless box $\psi_n = \sqrt{2} \sin(n\pi x)$ shows the same absence of diagonal preference. The normalized diagonal ratios $\langle \psi_n | x^n | \psi_n \rangle / \langle \psi_1 | x | \psi_1 \rangle = (1, 0.74, 0.59)$ are monotone decreasing, opposite to the target $(c_1, c_2, c_3)/c_1 = (1, 1.5, 8)$. The index matching of A296 is therefore not present in the position-moment structure of the bare eigenfunctions.*

This closes a fourth mechanism class. With A292 (universal first order), A295 (dressing, wrong κ -order), A296 (external diagonal probe), and the present position-moment reading all excluded, the surviving requirement of A296 stands unaltered: the coupling must join level n to degree n at first order in κ with strength $\kappa \Omega / \lambda_1$.

2 Why the Amplitude Cannot Carry It

Proposition 2.1 (Linear onset at the center). *Each eigenfunction has leading power one as $x \rightarrow 0$: the fitted log-log slopes are $(0.998, 0.992, 0.985)$ for the three levels. This is forced, not fitted. The center is the point where observation vanishes (A281), and the boundary condition there is Dirichlet, $\psi(0) = 0$. A smooth potential then gives every level the same linear onset, so the degree of the leading term at the origin is one for all n and cannot encode n .*

Remark 2.2 (The feature is nodal). *What distinguishes level n is not its shape at the center but the number of times it crosses zero: ψ_n has $n - 1$ interior nodes. The author's reading, recorded as a reading and not a result, is that this is the natural home of the rule. The center must be a zero for every family, so the families cannot differ there; they can differ only in how many further zeros they carry, and "family n draws degree n " should then be a statement about the n -th nodal sector rather than about the amplitude. The next probe (a separate pre-registration) tests whether a nodal or derivative pairing reproduces the coefficients $(2, 3, 16)$ that the position moments do not.*

Status. OI-287-1 remains open. Its surviving form, after this addendum, is one sentence: *find the nodal or derivative pairing that couples the n -th level to the degree- n monomial at strength $\kappa \Omega / \lambda_1$.* Nothing here changes a published number; the lepton law and its 1×10^{-4} accuracy (A287) are untouched.

Verifier. `verify_P297.py`: rebuilds H , recomputes A_{nk} and the box control, and asserts the absence of diagonal preference, the opposed diagonal ratios, and the unit leading powers. All PASS.