

Addendum 291: The Ergodic Dressing

Link 2 proven in the limit, selected by the data:
the α chain completes (Q1 of the closure program)

TOE Corpus

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Skill: gap-closure v1.1 — **Outcome:** Case A (exact limit theorem + decisive data selection; residual tension owned with direction) — **Closes:** OI-283-1 — **Verifier:** `verify_P291.py`
— all PASS

Abstract

A288's chain for the α -comma had one soft link: that the oscillation's variance is apportioned across the layers by their measure fractions. The link is here computed, and the computation is richer than the assumption. For the n -th perturbation mode $\sin(n\pi x)$ of P03's wave equation, the layer-projected energies have closed forms:

$$E_{\text{edge}}(n) = \frac{\pi}{2}, \quad E_{\text{bnd}}(n) = \frac{\pi^2}{2} - \frac{3}{4n^2}, \quad E_{\text{blk}}(n) = 2\pi^3 - \frac{6\pi}{n^2},$$

(the edge integral is n -independent: the edge hears every mode equally). *Theorem 1.2:* the shares converge as $1/n^2$ to $(2\pi^3, \pi^2/2, \pi/2)/(\Omega/2) = (4\pi^3, \pi^2, \pi)/\Omega$ — **exactly the measure shares** — so Link 2 holds identically in the spectrum-ergodic limit, and fails for any fundamental-dominated ensemble (the $n=1$ edge share is 0.03211, 40% high). The data then choose: the required unobserved share is $x = 0.022737$ (CODATA-22) against the ergodic $\pi/\Omega = 0.022925$ (deviation 1.9×10^{-4}) and the fundamental's 0.03211 (deviation 9.4×10^{-3}): **the ergodic reading is selected at $31\times$ the CODATA edition spread**. The ergodic ensemble is, moreover, the corpus's own operating mode: P29's exposure principle (states sample the fixed architecture's full spectrum) and A274's unique ergodicity supply it; concentration in the ground mode is what *training* would produce, and the kernel does not train. Direction analysis closes the loop: the $1/n^2$ corrections are *positive* (any mode mixture raises the edge share above π/Ω), while the residual $2.8\sigma_{\text{exp}}$ tension requires a *negative* shift — so no ensemble explains it, the strict limit is the unique prediction, and CODATA-26 will move toward 137.035999236 or kill the chain. The α chain now reads, with no soft links: P03 oscillation (amplitude κ , canon) $\rightarrow$ ergodic layer shares (Theorem 1.2, exact) $\rightarrow$ edge unobserved (A281, derived) $\rightarrow$ linear average (A283, exact; exponent data-selected) $\Rightarrow \alpha^{-1} = \Omega\sqrt{1 - \kappa^2(1 - \pi/\Omega)}$. OI-283-1 closed; Q1 of the closure program complete; P39 (*The Fine-Structure Constant*) is now assemblable from A279, A283, A284, A288, and this addendum.

1 The Theorem

Proposition 1.1 (Closed forms). *For $n \geq 1$, with $\rho_{\text{edge}} = 2\pi x$, $\rho_{\text{bnd}} = 3\pi^2 x^2$, $\rho_{\text{blk}} = 16\pi^3 x^3$: $\int_0^1 \rho_\ell(x) \sin^2(n\pi x) dx$ equals $\pi/2$, $\pi^2/2 - 3/(4n^2)$, and $2\pi^3 - 6\pi/n^2$ respectively (verified symbolically and by quadrature at 40 digits). The total is $\Omega/2 - (6\pi + 3/4)/n^2$.*

Theorem 1.2 (Ergodic equipartition). *The layer shares of mode n , $s_\ell(n) = E_\ell(n)/\sum_m E_m(n)$, satisfy*

$$s_\ell(n) = f_\ell(1 + O(1/n^2)) \xrightarrow{n \rightarrow \infty} f_\ell = \frac{(4\pi^3, \pi^2, \pi)}{\Omega}, \quad (1)$$

with the limit equality exact (algebra: $(\pi/2)/(\Omega/2) = \pi/\Omega$, etc.). Hence for any ensemble of modes whose weight is not concentrated at small n — in particular the exposure ensemble of P29 and the time-ergodic state of A274 — the oscillation’s variance is apportioned by layer measure: Link 2 of A288, previously an assumption, is the ergodic limit of an exact computation. For the fundamental alone: $s_{\text{edge}}(1) = 0.03211$, $s_{\text{bnd}}(1) = 0.08555$, $s_{\text{blk}}(1) = 0.88234$ — equipartition fails at 40% in the edge channel; the assumption was never trivially true, which is what makes its selection by the data (next section) evidence rather than bookkeeping.

2 The Data Select

Proposition 2.1 (Selection at $31\times$ the edition spread). *Inverting the measured deficits gives the required unobserved share $x = 0.022439$ (CODATA-18) and 0.022737 (CODATA-22); edition spread 3.0×10^{-4} . Candidates: ergodic limit $\pi/\Omega = 0.022925$ (deviation $+1.9 \times 10^{-4}$ from CODATA-22, inside one spread); fundamental-only 0.03211 (deviation $+9.4 \times 10^{-3}$, $31\times$ the spread; excluded). The measurement is decisively in the ergodic regime.*

Remark 2.2 (Direction: the residual cannot be mode mixture). $s_{\text{edge}}(n) = f_{\text{edge}}(1 + 0.286/n^2 + \dots)$: *every finite- n admixture pushes the edge share up. The standing $2.8\sigma_{\text{exp}}$ residual requires x below f_{edge} . Therefore no spectral ensemble closes the residual; the strict ergodic limit is the chain’s unique prediction, the tension is irreducible within the mechanism, and the test is wholly external: future CODATA editions move toward $\Omega\sqrt{1 - \kappa^2(1 - \pi/\Omega)} = 137.035999236$ or the chain dies (A284’s direction test, now sharpened: there is no second knob).*

Remark 2.3 (Why ergodic is the corpus’s reading, not a choice). *Exposure (P29, Pin 4): the kernel’s states select trajectories through a fixed architecture — the full spectrum is sampled, none trained into dominance. Unique ergodicity (A274): time averages equal space averages along the breath. A fundamental-dominated dressing would describe a system that has learned its ground state — exactly what the Training/Exposure theorem forbids the kernel from doing. The data’s $31\times$ preference for the ergodic share is thus also an experimental echo of Pin 4: the first time the exposure-not-training principle has been weighed by a precision measurement and preferred.*

Consequences. OI-283-1 closed. The α chain is complete with no soft links: oscillation (canon) $\rightarrow$ equipartition (theorem, ergodic) $\rightarrow$ edge exclusion (derived) $\rightarrow$ average (exact) $\rightarrow \alpha^{-1} = \Omega\sqrt{1 - \kappa^2(1 - \pi/\Omega)}$, parameter-free, at 0.43 ppb from CODATA-22 with a one-way falsification direction. Q1 of the closure program is complete; P39 is assemblable. Registry note for P000_1_c updated by the verifier.

Verifier. `verify_P291.py`: the closed forms (symbolic vs quadrature, 40 digits, $n = 1, 2, 5, 50$); the exact limit identity; the $1/n^2$ rate and its coefficient; the share table; the data inversion and the $31\times$ selection; the direction statement; the chain-status assertions. All PASS.