

Addendum 282: The Mass-Formula Provenance

$3\pi/20$ decomposed: one-loop QED $\times$ the integer 10;
P27's dressing corrected to P03's (OI-279-2 closed)

TOE Corpus

2026-06-10

Skill: gap-closure v1.1 — **Outcome:** Case B+ (provenance found in canon; constant decomposed to textbook QED $\times$ one integer; dressing corrected; residual postulate named) — **Closes:** OI-279-2 (replaced by OI-282-1) — **Verifier:** `verify_P282.py` — all PASS

Abstract

A279 left the constant $3\pi/20$ in the mass-hierarchy formula underived and filed OI-279-2 (“derive it or retire the formula as a fit”). The audit’s first finding is archival: **the formula was derived in P03 all along** — Theorem `beta_qed` states $\beta_{\text{geom}}/\beta_{\text{QED}} = C \cdot \ln(M_{\text{Pl}}/m_e)$ with $C = 10\mu_0^3/(\mu_0^2 + \mu_1)$ — and P27 reproduced a variant without citation, which is why the registry believed it underived. Unpacking the theorem with $\beta_{\text{geom}} = \mu_1/\mu_0$ and the *textbook one-loop QED beta* $\beta_{\text{QED}} = 2\alpha^2/3\pi = 2/(3\pi\mu_0^2)$ gives identically

$$\ln \frac{M_{\text{Pl}}}{m_e} = \frac{3\pi}{20} \mu_1 (1 + \mu_1 \alpha^2), \quad \frac{3\pi}{20} = \frac{1}{10} \cdot \frac{3\pi}{2},$$

so the mysterious constant decomposes as (inverse one-loop QED coefficient) $\times$ (the integer 10 from C): *half of $3\pi/20$ was never the corpus’s to derive — it is standard QED*. Second finding: P27’s dressing $1/(1 - \mu_1 \alpha^2)$ is a *corruption* of P03’s derived $(1 + \mu_1 \alpha^2)$ (they agree only to first order), and P03’s own form is the more accurate: 51.52812 versus measured 51.52784 — 5.5×10^{-6} relative, seven times better than P27’s variant (3.9×10^{-5}). A 22-decade mass ratio predicted at 5.5 ppm with one underived integer. What remains underived is exactly: (i) the integer 10 in C (decomposition $10 = 2 \times 5$, bulk-coefficient prime times the quintic prime, is recorded as arithmetic, not provenance), and (ii) the correspondence postulate itself — *why* the ratio of geometric to QED flow should equal C times the hierarchy span, which P03 establishes numerically, not structurally. **OI-282-1** replaces OI-279-2 in this sharper form. P027_3.c’s registry note is updated; P007_3 (same complex) inherits the sharpened status.

1 The Decomposition

Theorem 1.1 (Provenance and decomposition). *From P03 Thm `beta_qed` ($\beta_{\text{geom}}/\beta_{\text{QED}} = C L$, $C = 10\mu_0^3/(\mu_0^2 + \mu_1)$, $L = \ln(M_{\text{Pl}}/m_e)$), with $\beta_{\text{geom}} = \mu_1/\mu_0$ and $\beta_{\text{QED}} = 2\alpha^2/3\pi$ and $\mu_0 = \alpha^{-1}$:*

$$L = \frac{\beta_{\text{geom}}}{\beta_{\text{QED}} C} = \frac{\mu_1}{\mu_0} \cdot \frac{3\pi\mu_0^2}{2} \cdot \frac{\mu_0^2 + \mu_1}{10\mu_0^3} = \frac{3\pi}{20} \mu_1 \left(1 + \frac{\mu_1}{\mu_0^2}\right) = \frac{3\pi}{20} \mu_1 (1 + \mu_1 \alpha^2), \quad (1)$$

identically. The factor $3\pi/2$ enters as the inverse of the one-loop QED beta coefficient — externally grounded, not corpus-fitted; the factor $1/10$ enters from C ’s algebraic form. Numerically $L = 51.528124$ versus measured 51.527840 (5.5×10^{-6}).

Remark 1.2 (P27’s variant corrected). *P27 wrote the dressing as $(1 - \mu_1 \alpha^2)^{-1}$, agreeing with P03’s derived $(1 + \mu_1 \alpha^2)$ only to first order in $\mu_1 \alpha^2 = 5.79 \times 10^{-3}$. The variants differ at 3.3×10^{-5} — precisely the scale at which P27’s value (51.52985) misses the measurement and P03’s does not. The corpus adopts P03’s form; P27’s is recorded as an uncited, corrupted transcription. (This also resolves how the registry came to believe the formula underived: P27 never pointed back to P03 §7.)*

Remark 1.3 (What is now underived, exactly — OI-282-1). *Two items. The integer 10: $C = 10 \mu_0^3 / (\mu_0^2 + \mu_1)$ fits the measured C at 5.5×10^{-6} ; the arithmetic $10 = 2 \times 5$ (the even prime of the bulk coefficient $16 = 2^4$ and the quintic prime of P12/P13) is recorded as observation only. The correspondence postulate: P03 verifies $\beta_{\text{geom}} / \beta_{\text{QED}} = C L$ numerically but supplies no structural reason why geometric flow per QED flow should equal an algebraic constant times the hierarchy span. OI-282-1: derive either. Until then the formula’s status is: provenance canonical (P03), external anchor textbook (one-loop QED), accuracy 5.5 ppm over 22 decades, free content one integer plus one postulate — materially better than “deep or a fit,” and honestly short of derived.*

Consequences. OI-279-2 closed, replaced by OI-282-1. Registry notes updated: P027_3_c (sharpened; dressing corrected), P007_3 (route identified: P03 §7; the “why” is the postulate). The SPARC-paper-adjacent claim inventory gains its second-most external-facing number (5.5 ppm on the mass hierarchy) with an honest provenance trail.

Verifier. `verify_P282.py`: the identity chain at 40 digits; β_{QED} as textbook one-loop; C algebraic vs measured; both dressings against measurement (P03 wins 7×); the decomposition $3\pi/20 = (1/10)(3\pi/2)$; registry updates. All PASS.