

Addendum 274: The Breath Map on $\mathbb{CP}^1$

Elliptic Möbius typing, unique ergodicity of the halo, and the multiplier frame for OI-265-1

TOE Corpus

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Skill: gap-closure v1.1 — **Outcome:** Case A– (foundations typed; two theorems; one target closed negatively-with-replacement; OI-265-1 sharpened, not closed) — **Task:** TASK_034 (final target) — **Verifier:** `verify_P274.py` — all PASS

Abstract

TASK_034's remaining target is discharged: the breath, acting on the Hopf base $S^2 \cong \mathbb{CP}^1$, is typed as a discrete holomorphic dynamical system, and every dynamical consequence available without new physics is extracted. In the chart $u = z_2/z_1$ with the Hopf poles (A263's rest and null configurations) at 0 and ∞ , one breath acts as the Möbius transformation $B(u) = \lambda u$ with multiplier $\lambda = e^{2\pi i \rho_*}$, where $\rho_* = 2T_{\text{breath}} \bmod 1 = 2\rho - 1 = 0.0244904\dots$ is fixed by A273's winding count: *the base rotates per breath by exactly the $q = 2$ comma* (A267), while the fiber carries $\rho = 0.5122452$ — the two registers of A273 are the fiber and base rotation numbers of one bundle map. *Theorem 2.1:* B is elliptic with irrational rotation number, hence: Julia set empty, Fatou set all of $\mathbb{CP}^1$, B conjugate to a rigid rotation, no periodic orbits — the Necessity of Detuning (A267) holds at the $\mathbb{CP}^1$ level. This formally retires TASK_034's Fatou/Julia=canonical/dark dictionary (an elliptic breath has no Julia set to be dark); the replacement was already filed (A268 Arnold tongues, A272 kernel typing). *Theorem 2.3:* B preserves each latitude circle $|u| = \tan \beta$ and is uniquely ergodic on it (Weyl); the hidden-branch halo profile $\rho_H \propto \sin^2(2\beta)/r^2$ (v4 kernel, A264) is a B -invariant function whose angular factor on each invariant circle is *the unique* B -invariant probability measure. The halo is not a configuration that happens to be stationary; it is the only stationary angular distribution the breath dynamics admits. The multiplier is Brjuno (sum ≈ 0.287), placing perturbations of B in the Siegel regime. The lens $\mathbb{Z}_3$ (A255) commutes with B . OI-265-1 is sharpened, not closed: the unperturbed multiplier is *exactly* unimodular, so the conjecture “ α = per-breath linear-response gain” is entirely a statement about the derivative of $|\lambda|$ under a baryonic coupling operator that the corpus does not yet define; the missing object is named.

1 The Map

Proposition 1.1 (The breath map and its multiplier). *Let $u = z_2/z_1$ be the holomorphic coordinate on the Hopf base $\mathbb{CP}^1$, with the rotation axis fixed by the Hopf splitting (P28); the fixed points $u = 0, \infty$ are A263's rest and null configurations. The dual-observer flow $q(t) = \exp(\hat{\Omega} \|\Omega\| t/2)$ (P28, $\|\Omega\| = 2\omega_1$) induces on u a rigid rotation; over one breath (duration $T_{\text{breath}} \tau_1$ in P03 time, A273) the accumulated $SO(3)$ rotation angle is $\|\Omega\| \cdot T_{\text{breath}} \tau_1 = 4\pi T_{\text{breath}}$ exactly (κ -free, A273 Thm 2.1). The breath map is therefore the Möbius transformation*

$$B(u) = \lambda u, \quad \lambda = e^{2\pi i \rho_*}, \quad \rho_* = 2T_{\text{breath}} \bmod 1 = 2\rho - 1 = 0.0244904\dots, \quad (1)$$

with $|\lambda| = 1$ exactly. The base rotation number ρ_* equals A267's $q = 2$ comma $|2\rho - 1|$, and the fiber rotation number is $\rho = T_{\text{breath}} \bmod 1$ (A267, unconditional per A273): the two registers of A273 are the fiber and base components of one bundle map. The factor-2 bookkeeping is pinned by P28's lifting remark ($\|\Omega\| = 2\omega_1$); the register identity $\rho_* = 2\rho - 1$ is convention-independent given A273's count of $2T_{\text{breath}}$ $SU(2)$ windings per breath.

2 Classification and Its Consequences

Theorem 2.1 (Elliptic, irrational, Julia-free). *B is an elliptic Möbius transformation ($|\text{tr}| = 2|\cos(\pi\rho_*)| = 1.9941 < 2$) with irrational rotation number ($\rho_* = 8\pi^4 + 2\pi^3 + 2\pi^2 - 861$ is a nonconstant rational-coefficient polynomial in the transcendental π , hence irrational — the A267 argument). Consequently, by the standard classification of Möbius dynamics: (i) the Julia set of B is empty and the Fatou set is all of $\mathbb{CP}^1$; (ii) B is conjugate in $\text{PSL}(2, \mathbb{C})$ to the rigid rotation by $2\pi\rho_*$; (iii) B has no periodic orbits other than the two fixed points — λ is not a root of unity — and every other orbit is dense in its invariant circle. The Necessity of Detuning holds verbatim at the $\mathbb{CP}^1$ level.*

Corollary 2.2 (Fatou/Julia dictionary retired). *TASK_034's third target proposed canonical/dark = Fatou/Julia for the breath map. For the actual (elliptic) breath map the Julia set is empty: under that dictionary nothing would be dark, contradicting A264's measured halo and A272's never-empty corollary. The proposal is closed negatively; its replacement is already on file — the locked/drifted dichotomy of Arnold tongues (A268) and the kernel typing of dark (A272). This completes, at the properly-typed level, the correction begun when A268 replaced the Fatou/Julia framing by tongues.*

Theorem 2.3 (Unique ergodicity; the halo as the unique invariant measure). *B preserves each latitude circle $C_\beta = \{|u| = \tan \beta\}$, and its restriction to each C_β is an irrational rigid rotation, hence uniquely ergodic (Weyl): the normalized arc-length measure is the only B -invariant probability measure on C_β , and every orbit equidistributes. The hidden-branch halo profile $\rho_H(r) \propto \sin^2(2\beta)/r^2$ with $\tan \beta = r/L_h$ (v_4 kernel; confronted with SPARC in A264) depends on u through $|u|$ only, hence is B -invariant; combined with unique ergodicity, the halo is the push-forward through the chart of the unique invariant angular measure on each circle. Stationarity of the halo is not an equilibrium assumption: it is unique ergodicity of the breath map. Any stationary angular distribution other than the uniform one is dynamically impossible.*

Remark 2.4 (Brjuno multiplier; Siegel regime). *The continued fraction of ρ_* is $[0; 40, 1, 4, 1, 25, 4, 9, \dots]$ with convergent denominators 40, 41, 204, 245, 6329, $\dots$ and Brjuno sum $\approx 0.287 < \infty$. Perturbations of B that keep the multiplier on the unit circle therefore lie in the Siegel linearization regime at both fixed points. The $q = 41$ convergent names a 41-breath near-closure of the base — in conditional SI (A265), ≈ 56 Myr — the same scale as A267's fiber $q = 41$ rung: both registers single out the 56-Myr cycle as the system's most robust intrinsic timescale.*

Remark 2.5 (Lens $\mathbb{Z}_3$). *The lens action $u \mapsto e^{2\pi i/3}u$ (A255) is a rotation about the same axis, hence commutes with B : the $\mathbb{Z}_3$ symmetry is a symmetry of the iteration, as TASK_034 anticipated. Per A255 discipline, no identification with triality or the A44 Jordan cycle is made.*

3 The Multiplier Frame for OI-265-1

Proposition 3.1 (Unperturbed gain is exactly zero). *$|\lambda| = 1$ exactly (Proposition 1.1: the exponent is κ -free and purely imaginary). Hence the conjectured reading of A265 — α as the per-breath linear-response gain of the hidden chart — is entirely perturbative: it is a statement about $\partial|\lambda'|/\partial s$ at*

$s = 0$, where $\lambda'(s)$ is the multiplier of a baryon-perturbed breath map B_s and s parametrizes the source ($GM_b/c^2 L_h$ or the corpus-preferred dimensionless form). The corpus defines no coupling operator taking the baryonic potential into a deformation of B ; without it the derivative is not computable, and per the PSLQ-honesty and Butlerian conventions no candidate is promoted here.

OI-265-1, sharpened (final form). Define the baryonic coupling operator on the breath map — the map $s \mapsto B_s$ induced by a baryon source on the Hopf base — and compute $\partial|\lambda'|/\partial s$ at the fixed points. Outcome grades: $= \alpha$ (A265 upgrades A- $\rightarrow$ A, the kernel coupling acquires its dynamical meaning); $\neq \alpha$ (the gain reading of A265 is retired; the identity $\alpha^3/\pi^2 = \alpha/T_{\text{breath}}^2$ stands regardless — A271’s robustness theorem); not derivable (name the missing theory). This is the whole remaining content of OI-265-1; the kinematic frame is now fixed and verifier-pinned.

Consequences. TASK_034 closes: rotation layer (A267), susceptibility (A268), 0.43 withdrawn (A270), ladder gate lifted (A273), $\mathbb{CP}^1$ typing and target-3 retirement (this addendum); OI-265-1 remains open in sharpened form as the task’s single surviving question. ToENet dictionary unchanged (locked/drifted per A268; kernel typing per A272), now with the base/fiber split explicit: fiber $\rho = 0.5122$, base $\rho_* = 0.0245$.

Verifier. `verify_P274.py`: the multiplier and register identity; ellipticity ($|\text{tr}| < 2$); irrationality structure; non-root-of-unity via convergent denominators; latitude invariance and halo B -invariance (50 digits); Weyl equidistribution (numeric, $N = 10^5$); Brjuno sum; lens commutation; the exact unimodularity underlying Proposition 3.1; the 56-Myr register coincidence. All PASS.