

Addendum 273: The Unit Bridge

The P03 $\leftrightarrow$ P14 conversion derived; Branch B excluded;
the breath ladder unconditional; TBS 28_2 retired

TOE Corpus

2026-06-10

Skill: gap-closure v1.1 — **Outcome:** Case A (bridge derived from canon text; exact cancellation theorem; branch decided) — **Closes:** OI-271-1; retires TBS 28_2 — **Verifier:** `verify_P273.py` — all PASS

Abstract

A271 sharpened TBS 28_2 to a missing unit bridge: the corpus carries P03's oscillation time (in which $\omega_1 = \pi\sqrt{1-\kappa}$) and P14's system units (in which $T_{\text{breath}} = \pi\alpha^{-1}$) with no documented conversion, leaving the windings-per-breath count two-valued (Branch A: T_{breath} ; Branch B: $T_{\text{breath}}\sqrt{1-\kappa}$). The bridge does not need to be chosen; it is already derived in the canon. P14's Breathing-Period theorem defines the breath by $\omega_{\text{breath}} = \omega_1/(\pi\alpha^{-1})$ — the system unit is the *fundamental oscillation period* $\tau_1 = 2\pi/\omega_1 = 2/\sqrt{1-\kappa}$ in P03 time (Proposition 1.1). Two consequences follow exactly. (1) *Cancellation* (Theorem 2.1): every per-breath count of an ω_1 -paced rate is κ -free, because the rate carries ω_1 and the unit carries ω_1^{-1} ; in particular the density register completes exactly T_{breath} cycles per breath and the dual-observer arc is exactly 4π per system unit. TBS 28_2 thereby dissolves: $\|\Omega\| = 2\pi\sqrt{1-\kappa}$ per P03 time *and* exactly 4π per system unit are the same speed in two units; nothing vanishes, nothing is broken, and the exactness is a unit conversion, not a dynamical closure — the Necessity of Detuning (A267) is untouched. (2) *Branch decision* (Theorem 3.1): Branch B's identification (P14 unit $\equiv$ P03 time) forces a breath of $T_{\text{breath}}/\tau_1 \approx 215.03$ system units, contradicting the Breathing-Period theorem; Branch B is excluded by the canon text. Branch A's *number* is confirmed at the density register — $\rho = T_{\text{breath}} \bmod 1 = 0.5122452$, definitionally exact — though its stated identification (“arc-radian time”) is replaced by the derived bridge. A267's ladder ($q = 2, 41, 449$) and tritone reading are hereby *unconditional*. A271's working hypothesis (tempered Branch B) is overturned; recorded per convention. Two closing identities: the SU(2) double-cover register carries $2T_{\text{breath}}$ windings per breath with rotation number $2\rho - 1 = 0.0244904$ — exactly A267's $q = 2$ comma (counting in the double cover *is* observing breath pairs); and $1 - \sqrt{1-\kappa} = \kappa/2 + O(\kappa^2)$ is retyped as a conversion residue, demoting A266's schisma near-miss flag.

1 The Bridge

Proposition 1.1 (The system unit is the fundamental period). *P14, Theorem (Breathing Period), defines the breath by $\omega_{\text{breath}} = \omega_1/(\pi\alpha^{-1})$ with $\omega_1 = \pi\sqrt{1-\kappa}$, “measured in fundamental oscillation units.” Hence the P14 system unit is one period of the tempered fundamental mode:*

$$1 \text{ system unit} = \tau_1 = \frac{2\pi}{\omega_1} = \frac{2}{\sqrt{1-\kappa}} = 2.0021362\dots \text{ (P03 time)}, \quad \tau_1\sqrt{1-\kappa} = 2 \text{ exactly.} \quad (1)$$

The conversion is tempered (it carries $\sqrt{1-\kappa}$) and it is derived, not chosen: it is read off the canon's own definition of the breath. The breath duration in P03 time is $T_{\text{breath}} \tau_1 = 2\pi\alpha^{-1}/\sqrt{1-\kappa} = 861.944\dots$

2 Cancellation

Theorem 2.1 (Per-breath counts are κ -free). *Let r be any rate proportional to ω_1 (a quantity paced by the fundamental mode). Its count per system unit, $r \tau_1 = r \cdot 2\pi/\omega_1$, and hence per breath, is independent of κ : the temper in the rate and the temper in the unit cancel identically. Instances: (i) density-register cycles per breath $= (\omega_1/2\pi) \cdot T_{\text{breath}} \tau_1 = T_{\text{breath}}$ exactly; (ii) dual-observer arc per system unit $= \|\Omega\| \tau_1 = 2\omega_1 \cdot 2\pi/\omega_1 = 4\pi$ exactly (P28 lifting, $\|\Omega\| = 2\omega_1$); (iii) $SU(2)$ windings per breath $= 2T_{\text{breath}}$ exactly.*

Proof. Immediate from Proposition 1.1: $r \tau_1 = 2\pi r/\omega_1$ and $r \propto \omega_1$. The instances substitute $r = \omega_1/2\pi$ and $r = 2\omega_1/2\pi$. Verified to 50 digits (verify_P273, S2). $\square$

Corollary 2.2 (TBS 28_2 retired). *The dual-observer arc-speed is $2\pi\sqrt{1-\kappa}$ per unit P03 time and exactly 4π per system unit. P28's derivation of $\|\Omega\| = 2\pi$ (in P28's clock, which is ω_1 -paced) and the lifting computation $2\pi\sqrt{1-\kappa}$ (in P03 coordinate time) are the same speed expressed in two units; the 0.107% "deviation" was a unit mismatch, not a physical discrepancy. Neither horn of TBS 28_2 holds: the $\sqrt{1-\kappa}$ correction does not vanish (it is real in P03 time), and no corrected formula is needed (both formulas are correct). The TBS is retired by conversion. Note the exactness of 4π per unit is definitional — the same class of exactness as the fundamental period being one unit — and therefore does not constitute an exact dynamical closure; A267's Necessity of Detuning is not violated. The dynamical detunings (ρ irrational, $G_1 \neq 0$) stand exactly where they were.*

3 Branch Decision

Theorem 3.1 (Branch B excluded; the density register carries T_{breath}). *Under the derived bridge: (i) Branch B's identification (P14 unit $\equiv$ P03 time) implies a breath of T_{breath} P03-time units $= T_{\text{breath}}/\tau_1 = 215.026\dots$ system units, contradicting P14's Breathing-Period theorem (T_{breath} fundamental cycles per breath); no register yields the count $T_{\text{breath}}\sqrt{1-\kappa}$. Branch B is excluded by canon text. (ii) The density register completes exactly T_{breath} cycles per breath (Theorem 2.1(i)), so the breath map's rotation number is*

$$\rho = T_{\text{breath}} \bmod 1 = \pi\alpha^{-1} - 430 = 0.5122452\dots \quad (2)$$

definitionally exact — Branch A's number, with its derivation corrected (the unit is the tempered fundamental period, not an abstract arc-radian time).

Corollary 3.2 (A267 unconditional). *A267's continued-fraction ladder ($q = 2, 41, 449$), the near-tritone reading of successive breaths, and the conditional-SI rungs ($\approx 2.7, 56$ Myr, conditional only on A265's SI calibration) are hereby unconditional: the branch-conditionality marker of A271 is lifted. A271's robustness theorem is unchanged (the a_0 chain, 432, and G_1 were branch-invariant and remain so — now also branch-decided), and its working hypothesis (Branch B) is overturned; A271 carries a note to this effect per the same-day convention.*

Remark 3.3 (The double-cover shadow is the $q = 2$ rung). *The $SU(2)$ register carries $2T_{\text{breath}}$ windings per breath (Theorem 2.1(iii)); its rotation number is $2T_{\text{breath}} \bmod 1 = 2\rho - 1 = 0.0244904\dots$*

— exactly A267’s $q = 2$ comma $|2\rho - 1|$. Counting in the double cover is observing breath pairs: the covering map sends the breath map to its own first near-closure, and the shadow’s convergent denominators $(40, 41, 204, 245, \dots)$ contain A267’s $q = 41$ rung. No new dynamics; one identity locking A267’s ladder to P28’s lifting remark.

Remark 3.4 (Retyping $1 - \sqrt{1 - \kappa}$; schisma flag demoted). $1 - \sqrt{1 - \kappa} = \kappa/2 + O(\kappa^2) = 0.0010670\dots$ is now typed: it is the per-unit conversion residue between the two clocks $(\tau_1/2 - 1)$, not the comma of any closure map — it measures the difference of two tick lengths, not a failure to close. A266’s flagged near-miss against the schisma $(3^8 \cdot 5/2^{15} - 1 = 0.0011292, \text{ ratio } 0.945)$ thereby loses its candidate referent and is demoted from flagged near-miss to numerical coincidence, unless future work gives the conversion residue a closure-map reading. A266’s quarter-comma/ G_1 flag is unaffected.

Consequences. OI-271-1 closed (Case A). TBS 28_2 retired in its final form — the canon’s oldest unresolved normalization was a missing unit conversion, now derived. A267 unconditional; TASK_034’s branch-conditional gate lifted. The working dictionary for ToENet and Kairos fixes $\rho = 0.5122452$ (density register) with SU(2) shadow 0.0244904.

Verifier. `verify_P273.py`: the bridge identity $\tau_1\sqrt{1 - \kappa} = 2$ at 50 digits; all three cancellation instances; the Branch-B contradiction factor; ρ and the recomputed ladder; the shadow identity $2T_{\text{breath}} \bmod 1 = 2\rho - 1$ and its convergents; the $\kappa/2$ expansion bound; the demotion ratio. All PASS.