

Addendum 272: Nullspace Behavior

Dark as kernel dynamics: one definition,
three instances, three inherited theorems

TOE Corpus

2026-06-10

Skill: gap-closure v1.1 — **Outcome:** Synthesis (definitions and corollaries only; no new computation; falsifiable typing) — **Verifier:** `verify_P272.py` — all PASS

Abstract

The corpus uses “dark” in three registers: dark matter (the gravitating, non-converting sector, A259–A264), dark luminosity (ToENet’s failed-traversal channel), and the detuned/drifted side of the closure dichotomies introduced in A266–A268 (commas; quasiperiodic orbits outside Arnold tongues). This addendum fixes one definition covering all three: *dark denotes the dynamics of the kernel of a closure map*. Each register is an instance: (I1) the fold map P_0 (Hopf pushforward) has $\ker P_0 =$ the charged sector — the dark halo is literally the fold’s nullspace, with dynamics; (I2) the just-lattice closure homomorphism has commas as its non-closure residue; (I3) the mode-locking condition $q\rho - p = 0$ is a null condition whose complement is the drifting (“dark”) orbit class. Three structure theorems already proven this cycle then attach as corollaries of the definition: the kernel is **sealed** (A261 superselection: no conversion back to the image), **paced** (A269: the kernel shares the image’s clock — nullspaces are synchronized dynamics, not frozen storage), and **never empty** (A267: irrational rotation forbids exact closure; dark is mandatory, not accidental). A typing remark separates the two cosmological darks: dark *matter* is kernel (what the fold annihilates from the base); dark *energy* is cokernel — P32’s corner residual $\Lambda_0 = 1 - \pi^2/32$ is precisely the measure of what the embedded B^4 fails to cover. Finally, ToENet’s luminosity channel acquires literal semantics: dark nodes are kernel states of the canonical-closure projection, and plateau detection is local kernel-rank estimation in the sense of A254’s collapse delta — the OS metabolism and the cosmological dark sector are the same mathematics. Scope: synthesis; the content is the shared vocabulary and the inheritance of theorems, nothing else is claimed.

1 Definition and Instances

Definition 1.1 (Dark = kernel dynamics). *Let P be a closure map of the corpus’s collapse–projection family ($\text{Co}P$, P27): a projection, quotient, or closure condition taking states to their canonical/closed form. Dark (relative to P) denotes the states and dynamics of $\ker P$: that which P annihilates from its image but which persists, evolving, outside it.*

Proposition 1.2 (Instances). *(I1) Fold. $P_0 =$ the Hopf pushforward $S^3 \rightarrow S^2$; $\ker P_0 =$ the fiber-charged sector, $\ell(\ell + 1)$ modes per level (A259 Prop. 1.1). By A261 and A264 this kernel gravitates without converting and its kinetic density is the observed halo: dark matter is $\ker(\text{fold})$ with dynamics. The dark fraction per level, $\ell/(\ell + 1) \rightarrow 1$, makes kernel dominance at depth a structural prediction, qualitatively consonant with a dark-dominated universe (stated as consonance, not derivation). (I2) Chord closure. The homomorphism from the 3-smooth lattice to the breath cycle*

(A266/A267); its failure-to-close residue at the nearest lattice point is the comma G_1 . Detuning is the dark of harmony. (I3) Locking. The null condition $q\rho_{\text{eff}} - p = 0$ defines the Arnold tongues (A268); orbits in the complement drift without closure — the dark of dynamics, and the working ToENet dictionary (canonical = locked, dark = drifting).

2 Inherited Theorems

Corollary 2.1 (Sealed). $\ker P_0$ cannot leak into the image: every canonical sector commutes with the Hopf charge and the fold map is exactly equivariant (A261). Darkness is dynamically protected — this is why dark matter is dark, restated as a property of kernels under the canonical $\hat{O}$.

Corollary 2.2 (Paced). $\ker P_0$ shares the image’s clock: the breathing spectrum descends from the charge-blind radial density (A269), so kernel dynamics is synchronized, not stagnant. Nullspaces breathe.

Corollary 2.3 (Never empty). The closure conditions of (I2) and (I3) are never exactly satisfied: ρ is irrational (A267), so commas are nonzero and some orbit class always drifts. A corpus with empty nullspaces is impossible; dark is mandatory.

Remark 2.4 (Kernel vs. cokernel: the two cosmological darks). Dark matter is kernel: annihilated by the fold, absent from the image, present in dynamics. Dark energy is cokernel: P32’s corner residual $\Lambda_0 = 1 - \pi^2/32$ measures what the embedding $B^4 \hookrightarrow [-1, 1]^4$ fails to cover — absent from the image for the dual reason. The pair (kernel, cokernel) thus types the two darks of cosmology as the two exactness defects of one map family. Recorded as typing, exact at the level of definitions; any dynamical consequence would require new work.

3 Semantics for ToENet

Proposition 3.1 (Dark luminosity = kernel states). ToENet’s luminosity channel (canonical/dark) is assigned the semantics of Definition 1.1: a dark entry is a traversal in the kernel of the canonical-closure projection (it ran, deposited a trajectory, and did not close). Plateau detection — dark density exceeding canonical density over a region — is then local estimation of kernel rank, prototype A254’s collapse delta $\delta_\Omega = \text{rank } \ker(H_0(A) \rightarrow H_0(X))$. The corollaries transfer: dark entries are sealed (never silently promoted — the Butlerian line as kernel protection), paced (indexed by the same breath ledger as canonical entries), and never empty (a school whose students never stall has stopped measuring). The OS’s metabolism and the cosmology’s dark sector are one formalism.

Scope and what would strain it. This addendum adds vocabulary and inheritance, not computation. The typing is falsifiable in its own register: a corpus “dark” phenomenon not expressible as kernel or cokernel of a CoP closure map would break the definition’s claim to coverage. Candidates to watch: the iPaper’s Nullity (pre-actualization) is naturally the kernel of the frame-installation map — consistent; Wheel heat (A261’s thermalized defect) is kernel *energy* after the kernel’s support is removed — consistent but worth a careful future typing. **OI-272-1:** formalize the CoP closure-map family so Definition 1.1 quantifies over a defined class rather than a list.

Verifier. `verify_P272.py`: instance recomputations (kernel rank $\ell(\ell + 1)$; G_1 ; tongue-edge $2\pi\delta$; dark fraction limit; Λ_0 as complement measure) and the corollary citations; all PASS.