

Addendum 269: Breath-Pacing as a Spectral Theorem

Why no subsystem can generate its own rhythm,
and the completion of the a_0 chain

TOE Corpus

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Skill: gap-closure v1.1 — **Outcome:** Case A (OI-268-1 closed; Pin 3 derived; A265 chain complete modulo the unit) — **Verifier:** `verify_P269.py` — all PASS

Abstract

A268 left one link between the galactic acceleration scale and pure canon: inference (a) of A265 — that the hidden sector is *breath-paced* — held constitutionally (Pin 3) but not as kernel theorem. This addendum derives it. The argument is spectral and rests on A261’s commutation table. The breath descends from the breathing spectrum of the radial density: P03 linearizes perturbations of $\rho(x)$ to obtain $\omega_n = n\pi\sqrt{1-\kappa}$, and P28 lifts the fundamental through the $SU(2)/SO(3)$ covering to the arc-speed $\|\Omega\| = 2\omega_1 \approx 2\pi$. The radial ODE contains no Hopf charge: by A261, every radial operator commutes with $\hat{Q}$, so the kernel Hamiltonian *separates*, $H = H_{\text{rad}} \oplus H_{\text{fiber}}$ with $H_{\text{fiber}} = -c_f \hat{Q}^2$. Fiber charge shifts sector energies but cancels in the *spacing* of radial breathing levels: the fundamental ω_1 is identical in every charge sector — verified symbolically and numerically (q -independence exact). Therefore the hidden (charged) sector oscillates, accumulates phase, and closes its winding cycle on exactly the visible sector’s period: there is one clock because there is one radial density and one operator (A218b). **Pin 3** — “no subsystem generates its own rhythm” — **upgrades from constitutional to derived**. Corollary: A265’s inference (a) closes, completing the chain $a_0 = \alpha c^2 / (T_{\text{breath}}^2 L_h) = \alpha^3 c^2 / (\pi^2 L_h)$ as a *pure-canon theorem with zero undetermined dimensionless constants* — the only remaining freedom is the unit length L_h , which every theory must carry and which rotation curves have measured at ≈ 0.98 kpc. Inherited residue: P28 TBS 28.2 (the $\sqrt{1-\kappa}$ factor, 0.107%, with A266’s schisma near-miss attached to exactly this number) now sits at the foundation of the chain and is promoted to the thread’s principal open refinement (OI-269-1).

1 The Theorem

Theorem 1.1 (Breath-pacing). *Let the hidden sector be the Hopf-charged subspace of the canonical $\hat{O}$ (A261), which is the unique admissible assembly (A218b). Then its phase dynamics is paced by the same fundamental as the visible sector’s:*

1. (One operator) *There is no second generator: assembly uniqueness forecloses sector-specific dynamics; both charge sectors evolve under restrictions of the same $\hat{O}$.*
2. (Charge-blind density) *The breathing spectrum $\omega_n = n\pi\sqrt{1-\kappa}$ (P03) is obtained by linearizing the radial density $\rho(x)$; by A261 Thm. 1.1 every radial operator commutes with $\hat{Q}$, so the linearized problem is identical in every charge sector.*

3. (Separability) $H = H_{\text{rad}} \oplus H_{\text{fiber}}$, $H_{\text{fiber}} = -c_f \hat{Q}^2$: fiber charge adds a constant offset per sector and cancels in level spacings. The breathing fundamental — hence the winding period, hence T_{breath} — is q -independent, exactly.

One density, one operator, one clock. A subsystem cannot generate its own rhythm because every rhythm in the kernel descends from the charge-blind radial spectrum: Pin 3 is a theorem.

Corollary 1.2 (The a_0 chain, complete). *A265 inference (a) closes; with A268's gain-unity result (inference (b)) the chain is fully derived:*

$$a_0 = \alpha \frac{c^2}{T_{\text{breath}}^2 L_h} = \frac{\alpha^3}{\pi^2} \frac{c^2}{L_h} = 1.1726 \times 10^{-10} \text{ m/s}^2 \text{ at } L_h = 0.978 \text{ kpc},$$

with zero undetermined dimensionless constants. The single dimensional freedom, L_h , is the unit any theory must carry — here measured by SPARC (A264) and quadruply locked (B^4 scale $\equiv$ fold-clock unit $\equiv$ amplitude length). Within the canon's own standards this is the strongest derivational chain it possesses: from the monad $\Omega = 4\pi^3 + \pi^2 + \pi$ to a number on a telescope, every link now a theorem, every theorem with a verifier.

Remark 1.3 (The residue moves to the foundation — OI-269-1). *The lift $\|\Omega\| = 2\pi\sqrt{1-\kappa}$ carries P28's TBS 28_2: a 0.107% deviation from exactly 2π , previously cosmetic, now foundational — it propagates (at second order, hence harmlessly today) into T_{breath} and thus a_0 . A266 flagged the schisma (0.113%) as a near-miss to exactly this number. Whether the $\sqrt{1-\kappa}$ factor is exact physics (the breath genuinely detuned from 2π — another comma), or cancels (P28's open question), becomes the chain's principal refinement. If it is real, the corpus's two flagged near-misses (G1/quarter-comma; 28_2/schisma) may be two rungs of one temperament — TASK_035 chain 3 inherits this as its sharpest target.*

Verifier. `verify_P269.py`: ω_1 exact; q -independence of the spacing across sectors; separability grounds (A261 commutators); the lift residue 0.107%; the complete chain re-derivation of a_0 ; zero-free-dimensionless audit; all PASS.