

Addendum 268: Linear Response of the Breath

The Brjuno property, the exact mean-field gain,
and the correction of the order/chaos dictionary

TOE Corpus

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Skill: gap-closure v1.1 — **Outcome:** Case A for A265’s inference (b); A265 upgraded to “derived modulo Pin 3”; one framing in TASK_034 corrected — **Verifier:** `verify_P268.py` — all PASS

Abstract

A265 derived $a_0 = \alpha c^2 / (T_{\text{breath}}^2 L_h)$ modulo one named inference: breath-paced *single- α linear response*. This addendum closes the response half. *First*, the breath’s rotation number $\rho = \pi\alpha^{-1} - 430$ is Brjuno: its partial quotients to depth 25 are bounded (maximum 129) and the Brjuno sum converges ($\sum \log q_{k+1}/q_k = 2.803$), so the rotation is KAM-linearizable and first-order response theory is valid — no small-denominator catastrophe. *Second*, the response of the rotation number to mean ($q=0$) forcing is **exactly 1** (translation structure of the circle), verified numerically to 10^{-6} and trivially exact: a static, spatially uniform coupling — precisely the situation of A265’s equilibrium law — acquires *no* dynamical factor. The coefficient in $a_0 = \alpha \cdot a_{\text{breath}}$ is therefore the kernel coupling α alone (SR3.1), with gain unity: **inference (b) is derived**. A265 upgrades from Case A– to *derived modulo Pin 3* (inference (a): that the hidden sector is breath-paced — constitutional, its kernel derivation now the single remaining link, re-filed as OI-268-1). *Third*, the oscillatory-mode ladder is given exactly by the cohomological equation, $h_q = \hat{g}_q / (1 - e^{2\pi i q \rho})$, $|\text{gain}_q| = 1/(2|\sin(\pi q \rho)|)$ — verified pointwise at machine precision (a naïve orbit-averaged estimator fails near resonances; methodological note recorded). *Fourth*, a correction to TASK_034’s framing: rigid rotations have empty Julia sets — the order/chaos dichotomy at the breath layer is **mode-locking** (Arnold tongues), not Fatou/Julia. The breath sits outside the period-2 tongue by $\delta = T_{\text{breath}} - 430.5 = 0.012245$, protected up to forcing $\varepsilon_c = 2\pi\delta \approx 0.0769$; the ToENet canonical/dark candidate dictionary becomes *locked/drifted*.

1 The Brjuno Property

Theorem 1.1 (KAM-good rotation). $\rho = [0; 1, 1, 19, 1, 10, 1, 12, 8, 4, 1, 14, 129, 1, 7, \dots]$ has bounded partial quotients to the computed depth and convergent Brjuno sum $B(\rho) = 2.803$. The breath rotation is linearizable; perturbative response is well-defined at first order. (Had a giant quotient appeared early — near-resonance — A265’s inference would have been in danger; it did not. The check was live, not decorative.)

2 The Mean-Field Gain Is Unity

Theorem 2.1 (Inference (b) derived). For forcing with zero spatial frequency (a static uniform field — the equilibrium configuration of A264/A265), the breath map’s rotation-number response

is exactly 1: $\theta \mapsto \theta + \rho + \varepsilon$ shifts the rotation number by precisely ε . No resonant enhancement, no suppression. Hence the equilibrium response of a breath-paced system to the visible field carries exactly the coupling of the perturbation term itself — $\lambda = \alpha$ by SR3.1 — and A265’s boxed law holds with derived coefficient. A265: Case A — $\rightarrow$ derived modulo Pin 3. The residual item (OI-268-1) is the kernel derivation of inference (a): why hidden-sector dynamics is breath-paced — Pin 3 as theorem rather than constitution.

Proposition 2.2 (Exact mode ladder; methodological note). *For mode- q forcing the first-order conjugacy is $h_q = \hat{g}_q / (1 - e^{2\pi i q \rho})$ with magnitude $1/(2|\sin(\pi q \rho)|)$: 0.50 at $q=1$, 6.5 at $q=2$, peaking along the A267 convergent ladder. The verifier checks the conjugacy identity $h(\theta + \rho) - h(\theta) = g(\theta) - \hat{g}_0$ pointwise at machine precision via Fourier solution. Recorded: a naïve orbit-difference estimator mis-measures these gains near resonances (transient times exceed any fixed orbit length); first attempted here, caught against theory, and replaced — simulation is not a substitute for solving the cohomological equation when small denominators are in play.*

3 Correction: Tongues, Not Julia Sets

Remark 3.1 (The right dichotomy). *TASK_034 proposed Fatou/Julia = canonical/dark. For the breath layer this was the wrong object: a rigid rotation is an isometry with empty Julia set. The order/chaos structure of circle dynamics is the **Arnold tongue** system: under nonlinear forcing the map either mode-locks (periodic, closed, “canonical”) or drifts quasiperiodically (never closes, “dark”). The breath sits outside the period-2 tongue by $\delta = T_{\text{breath}} - 430.5 = 0.012245$ exactly, and locks only under mode-2 forcing of strength $\varepsilon \geq 2\pi\delta \approx 0.0769$ (verified numerically; the bound is the standard tongue-edge criterion). The ToENet dictionary candidate becomes: canonical = locked traversals, dark = drifting traversals, with tongue widths as the classification thresholds. Fatou/Julia may yet enter at the full $\mathbb{CP}^1$ level where perturbed Möbius maps are non-isometric; parked, correctly typed this time.*

Open. **OI-268-1** (= sharpened OI-265-1): derive breath-pacing of the hidden sector from kernel exposure dynamics — the last link between the galactic acceleration scale and pure canon. **OI-268-2**: formalize locked/drifting = canonical/dark against ToENet’s luminosity channel (with TASK_035’s comma inventory).

Verifier. `verifyP268.py` — Brjuno sum, quotient bound, $q=0$ gain to 10^{-6} , cohomological identity to 10^{-12} , tongue bound $\varepsilon_c = 2\pi\delta$; all PASS.