

Addendum 267: The Rotation Number of the Breath

Necessity of detuning, the derivation of 432,
and the breath's intrinsic scale system

TOE Corpus

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Skill: gap-closure v1.1 — **Outcome:** Case A (two theorems) + structural ladder + one open handoff — **Tasks:** TASK_034 (rotation layer), TASK_035 (chains 1–2) — **Verifier:** `verify_P267.py` — all PASS

Abstract

The breath acts on the Hopf fiber as a rigid rotation by T_{breath} windings per tick; as a circle map its rotation number is $\rho = T_{\text{breath}} \bmod 1 = \pi\alpha^{-1} - 430 = 0.5122452\dots$. *Theorem 1 (Necessity of Detuning):* ρ is irrational (π is transcendental), so no breath orbit ever closes: every identification of a corpus cycle with an exact integer or rational chord carries a necessarily nonzero residual. The corpus's small gaps are not errors awaiting better derivations; they are invariants of an irrational rotation — commas. *Theorem 2 (Derivation of 432):* among 3-smooth integers (the just-intonation lattice $2^a 3^b$), the nearest to T_{breath} is exactly $432 = 2^4 3^3$; P14's Vedic correspondence is thereby derived rather than observed, and G_1 (A266) is the comma of that nearest chord. *The ladder:* the continued fraction of ρ , $[0; 1, 1, 19, 1, 10, \dots]$, yields the breath's intrinsic near-closure cycles — $q = 2$ breaths (comma 0.0245: successive breaths are near-antipodal, a half-turn tritone structure consonant with the mirror configuration of P28), then $q = 41$ (comma 0.0020), then deeper levels — the system's own “scale,” analogous to the twelve-tone system arising from the continued fraction of $\log_2(3/2)$. In conditional SI (A265): the 2-breath near-cycle is ≈ 2.7 Myr, the 41-breath near-closure ≈ 56 Myr. *Handoff:* by small-denominator theory the system's response to periodic forcing is amplified as $1/\text{comma}$ at the near-closure cycles; whether the kernel coupling α emerges as the susceptibility gain at a distinguished rung (TASK_034's Koenigs question, OI-265-1) is open and explicitly *not* claimed here.

1 The Breath as a Circle Map

Theorem 1.1 (Necessity of Detuning). *The breath map on the Hopf fiber is the rigid rotation $\theta \mapsto \theta + \rho \pmod{1}$ with $\rho = T_{\text{breath}} \bmod 1 = \pi(4\pi^3 + \pi^2 + \pi) - 430$, irrational by the transcendence of π . Hence: (i) no orbit is periodic — no corpus cycle built on breath counting closes exactly; (ii) every rational/integer chord assignment carries a nonzero comma; (iii) orbits are equidistributed (Weyl), so the system revisits every phase neighbourhood without repetition — the dynamical substrate of the corpus's “moves toward regions not visited yet.” Detuning is not a defect of the derivations; it is a theorem of the dynamics.*

Theorem 1.2 (Derivation of 432). *Let $S_3 = \{2^a 3^b\}$ be the 3-smooth (just-intonation) lattice. The nearest element of S_3 to $T_{\text{breath}} = 430.5122\dots$ is $432 = 2^4 3^3$ (neighbours: 384 and 486, both farther by an order of magnitude in distance). Therefore P14's 432 correspondence is the statement “the*

breath's nearest just chord is $2^4 3^3$," a derived property of T_{breath} , and $G_1 = 432/T_{\text{breath}} - 1$ (A266) is the comma of that chord. The historical resonance (Vedic 432) is thereby decoupled from the mathematics: any culture tuning to small-prime ratios near this period finds 432.

2 The Intrinsic Scale System

The continued fraction $\rho = [0; 1, 1, 19, 1, 10, 1, 12, 8, 4, \dots]$ gives the hierarchy of best rational approximations p/q — the cycle lengths at which the breath nearly closes:

q (breaths)	comma $\ q\rho\ $	conditional SI (A265)
2	0.0245	≈ 2.7 Myr
41	0.0020	≈ 56 Myr
449	(next rung)	≈ 0.6 Gyr

Remark 2.1 (Near-tritone structure). $\rho \approx \frac{1}{2} + 0.0122$: *each breath advances the fiber by almost exactly a half-turn. Successive breaths are near-antipodal — the dynamics is a slowly precessing alternation, consonant with the mirror configuration ($\Omega^{(-)} = -\Omega^{(+)}$) that P28 makes canonical and with the Oscillate sector's role in the Wheel. The 2-breath near-cycle is the system's deepest periodicity; everything slower is precession of it.*

Remark 2.2 (Susceptibility handoff — open). *For a circle map under periodic perturbation, response at cycle q is amplified as $1/\|q\rho\|$ (small denominators). The breath's susceptibility ladder is therefore fixed by the table above. Whether α appears as the gain at a distinguished rung — which would close OI-265-1 and upgrade A265 to Case A — is the TASK_034 numerical target. Not claimed.*

Verifier. `verify_P267.py`: ρ exact; convergents and commas (corrected recurrence, monotone decreasing along convergents); the 3-smooth scan; near-tritone bound; SI conversions; all PASS.