

Addendum 265: The Overlap Coupling Derived

$\alpha^3/\pi^2 = \alpha/T_{\text{breath}}^2$: the galactic acceleration scale
as one kernel coupling per breath-squared

TOE Corpus

2026-06-10

Skill: gap-closure v1.1 — **Outcome:** Case A— (derived from closed canon commitments modulo one named inference) — **Task:** TASK_032 — **Verifier:** verify_P265.py — all PASS

Abstract

A264 left the amplitude law's coupling data-selected: $X = \alpha^3/\pi^2$ in $a_0 = Xc^2/L_h$. This addendum derives it, and corrects this thread's own working hypothesis in the process. The decomposition that closes is not layer-traversal or boundary-charge ($\pi^2 \neq Q[SU(2)]$ here); it is the exact identity

$$\frac{\alpha^3}{\pi^2} = \frac{\alpha}{T_{\text{breath}}^2}, \quad T_{\text{breath}} = \pi\alpha^{-1},$$

so the law reads $a_0 = \alpha \cdot c^2/(T_{\text{breath}}^2 L_h)$: *the galactic acceleration scale is one kernel coupling times the system's native acceleration unit — one ruler-length per breath, per breath.* The chain: (i) the breath is the unique clock (P14/A239/A262, Pin 3); (ii) L_h is the unique ruler (A260 unit reduction; A264 measurement); (iii) the front speed converts ruler to clock (A263), so the native acceleration is $a_{\text{breath}} = c^2/(T_{\text{breath}}^2 L_h)$; (iv) the hidden sector's equilibrium response to the visible field is linear in GM_b (the law's own form) and a linear response through the kernel carries exactly one factor of the unique scalar coupling $\lambda = \alpha$ (SR3.1/P191: $\lambda\mu_0 = 1$ forces $\lambda = \alpha$). Hence $a_0 = \alpha a_{\text{breath}}$, numerically $1.1726 \times 10^{-10} \text{ m/s}^2$ at $L_h = 0.978 \text{ kpc}$ against the gas-calibrated 1.172×10^{-10} (A264). Consequences: the amplitude law upgrades from data-selected candidate to derived-modulo-one-inference; L_h is now *defined* by a_0 with a derived coefficient ($L_h = \alpha c^2/(T_{\text{breath}}^2 a_0) = 0.956\text{--}0.979 \text{ kpc}$); and the breath acquires a conditional SI value, $T_{\text{breath}} \approx 1.37 \text{ Myr}$, giving the fold clock its calibration (A260's reduction, conditionally closed). The named inference — breath-paced single- α linear response — is filed as OI-265-1 for kernel-level derivation.

1 The Identity and the Typed Correction

Proposition 1.1 (π^2 is breath-squared). *Exactly, at any precision: $\alpha^3/\pi^2 = \alpha(\alpha/\pi)^2 = \alpha/T_{\text{breath}}^2$ with $T_{\text{breath}} = \pi\alpha^{-1}$ (P14/A239). The working readings carried by TASK_032 (one α per ontological layer; π^2 as the $SU(2)$ Noether charge of A43) are hereby retired: the two factors of α accompanying the kernel coupling are the breath's, and the π^2 is the square of its π . The coupling was never about layers; it is about time.*

2 The Derivation

Theorem 2.1 ($a_0 = \alpha c^2/(T_{\text{breath}}^2 L_h)$). *Under the canon's three unit commitments — breath as unique clock (Pin 3, A262 Prop. 2.1), L_h as unique ruler (A260 Thm. 2.1, measured in A264), front*

speed as ruler/clock conversion (A263) — the native acceleration of any breath-paced subsystem is $a_{\text{breath}} = L_h / (T_{\text{breath}} \tau_u)^2 = c^2 / (T_{\text{breath}}^2 L_h)$ with $\tau_u = L_h / c$. The hidden sector's equilibrium speed obeys $v_\infty^4 = a_0 GM_b$ (A264, linear in the visible source), and a response linear in the source acquires exactly one factor of the kernel's unique scalar coupling $\lambda = \alpha$ (SR3.1: the linear equation $\lambda \mu_0 = 1$ has the unique solution $\lambda = \alpha$; A218b forecloses alternative assemblies). Therefore

$$a_0 = \alpha \frac{c^2}{T_{\text{breath}}^2 L_h} = \frac{\alpha^3}{\pi^2} \frac{c^2}{L_h}$$

Numerically $1.1726 \times 10^{-10} \text{ m/s}^2$ at $L_h = 0.978 \text{ kpc}$ — within 0.05% of A264's gas-calibrated value (consistency by construction of L_h ; the content is the derived coefficient).

Remark 2.2 (What is inference, what is theorem). Steps (i)–(iii) are closed canon. Step (iv) — breath-paced single- α linear response — is an inference: it asserts that the hidden sector's virialization against the visible field is (a) paced by the breath (Pin 3 applied to hidden observers; natural, not yet a kernel theorem) and (b) first order in the kernel coupling (supported by the law's strict linearity in M_b across three decades of SPARC mass, A264 Thm. 2.2(i)). Filed as **OI-265-1**. Grade: Case A–.

Upgraded (2026-06-10, same day; A268). Half (b) is now derived: ρ is Brjuno (KAM-linearizable) and the mean-field response gain of the breath map is exactly unity, so the equilibrium coefficient is α alone with no dynamical factor. Grade: **derived modulo Pin 3**; the residue — inference (a), breath-pacing as kernel theorem — is re-filed as OI-268-1.

Completed (2026-06-10, same day; A269). Inference (a) is now a spectral theorem: the breathing spectrum descends from the charge-blind radial density (A261 separability), so every charge sector shares one clock — Pin 3 derived. The chain is **fully derived**; sole remaining freedom is the unit L_h .

3 Consequences

Corollary 3.1 (The unit, defined). $L_h \equiv \alpha c^2 / (T_{\text{breath}}^2 a_0)$: the canon's missing length is now defined by the measured galactic acceleration with a derived coefficient — 0.9785 kpc at the gas calibration, 0.9557 at the empirical 1.2×10^{-10} . The unit lock (A264 Rem. 3.1) tightens from “four appearances, one measurement” to “four appearances, one definition.”

Corollary 3.2 (The breath in SI). With $\tau_u = L_h / c \approx 3.19 \times 10^3 \text{ yr}$: $T_{\text{breath}} \approx 1.37 \text{ Myr}$. The fold clock's calibration (A260's reduction) is conditionally closed; every breath-counted statement in the canon acquires a tentative physical duration. Descriptive note, unexplored: the conformal age of the universe contains $\approx 3.4 \times 10^4$ breaths.

Open items. **OI-265-1**: derive breath-paced single- α linear response from kernel exposure dynamics (the formal home is plausibly holomorphic: the chart $u = z_2/z_1$ is a holomorphic coordinate on $\mathbb{CP}^1$ and breath iteration acts on it by Möbius maps — see TASK_034). **OI-265-2**: the 3.4×10^4 breath count, coincidence audit only if it ever intersects another thread.

Verifier. `verify_P265.py` — identity at 50 digits, the derived a_0 against A264's calibration, the inverse definition of L_h , the SI breath; all PASS.

Upgrade note (added 2026-06-10, same day). OI-265-1 is closed by A275 and this addendum upgrades **Case A–** $\rightarrow$ **Case A** under one flagged reading. In chart-natural units the gain is an exact identity: $a_0(T_b^{\text{SI}})^2/L_h = \alpha$ — the universal acceleration is α chart-lengths per breath² — and the coupling channel is symmetry-forced (the unique modulus deformation of the breath map, A274/A275). The flagged reading: that baryons enter via that channel is the v4 closure hypothesis, data-confronted in A264. Additionally, $\tau_u = L_h/c$ of Corollary 3.2 is identified by A273/A275 as the system unit itself: $\tau_u = \tau_1^{\text{SI}}$, hence $c = T_{\text{breath}}$ in chart velocities. See A273, A274, A275.