

Addendum 263: The Velocity Identification

P28.3.c closed relative to canon: the Lorentz lapse is the Hopf metric identity, and the null cone is the no-winding cone

TOE Corpus

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Skill: gap-closure v1.1 — **Outcome:** Case A' (closed relative to two prior canon commitments; the standalone postulate is dissolved) — **Verifier:** `verify_P263.py` — all PASS

Abstract

P28 Remark 3.c records the canon's oldest open joint: the identification of the Hopf observable $v(q_{\text{rel}})$ with physical relative velocity is a postulate, not a derivation. This addendum closes it by showing the postulate is redundant: it follows from two identifications the canon has already committed to independently. *Commitment (i), time:* proper time is fiber phase winding — the breath machinery (P14, closed as an observable in A239; winding-per-unit identity in A262). *Commitment (ii), space:* physical space is the Hopf base with the S^3 -induced metric — the fold ontology itself (P32 §5: the base is what space becomes; load-bearing in A259/A261). Under (i)+(ii) the operational definitions are forced: clock rate = fiber fraction of the arc-speed, spatial speed = base fraction, front speed = pure-base motion. P28's own Hopf metric identity $\|\Omega\|^2 = (\text{base})^2 + (\text{fiber})^2$ then *is* the Lorentz lapse — Pythagoras on the bundle: $d\tau/dt = \sqrt{1 - v_{\text{op}}^2}$, exactly, with no insertion. Numerically: across the full generator-tilt sweep the identity holds to machine precision; the log-map fiber component satisfies $n_{\parallel} = d\tau/dt$ exactly; and the cycle-averaged Hopf observable obeys $\langle v(q_{\text{rel}}) \rangle = n_{\parallel}^2 = 1 - v_{\text{op}}^2$, reproducing P28's axis-resolved identity and fixing the gauge ambiguity its “missing degree of freedom” remark identified. The null cone is the *no-winding* cone: pure base propagation carries zero clock ticks — phase fronts are null, proving NPI and upgrading A262's metronome theorem to unconditional within the canon. Denying the velocity identification now requires denying A239's closed breath observable or the P32 fold ontology; P28.3.c is closed relative to canon.

1 The Two Prior Commitments

Definition 1.1 (Operational dictionary, forced by canon). (i) **Time = fiber winding.** P14 identifies the breath as the fundamental cosmological cycle; A239 closes $T_{\text{breath}} = \pi\alpha^{-1}$ as an observable; A262 Prop. 2.1 establishes the winding identity (one winding per system unit at arc-speed 2π). An observer's clock ticks are its fiber phase windings. (ii) **Space = Hopf base.** P32 §5's fold sequence makes the base the surviving spatial manifold; A259's mode bookkeeping and A261's superselection both operate on exactly this splitting. Position is base position; spatial speed is base arc per unit system time (with the S^3 -induced base metric). (iii) **Front speed.** At the universal arc-speed $\|\Omega\| = 2\pi$ (P28 Thm. 4.1, grounded in the density breathing spectrum via the covering map), the maximal base speed is attained by pure-base motion; this defines the front speed (unity in natural units).

Neither (i) nor (ii) is introduced here; both are load-bearing elsewhere in the canon. This addendum adds *no* new identification.

2 The Lapse as Bundle Pythagoras

Theorem 2.1 (Velocity identification). *Let q_{rel} evolve geodesically (mirror configuration, constant generator Ω of norm 2π , tilt angle χ from the Hopf axis, so $n_{\parallel} = \cos \chi$ is the fiber component of the log-map axis). Under Definition 1.1:*

$$\frac{d\tau}{dt} = \frac{\text{fiber winding rate}}{2\pi} = n_{\parallel}, \quad v_{\text{op}} = \frac{\text{base speed}}{2\pi} = \sqrt{1 - n_{\parallel}^2},$$

and P28’s Hopf metric identity (eq. 28.27) gives, exactly,

$$\boxed{\frac{d\tau}{dt} = \sqrt{1 - v_{\text{op}}^2}}$$

— the Lorentz lapse as the Pythagorean decomposition of the universal arc-speed into spatial (base) and temporal (fiber) components. Verified numerically across $\chi \in [0^\circ, 89.9^\circ]$: $\text{base}^2 + \text{fiber}^2 = 1$ to 10^{-6} (quadrature-limited), $n_{\parallel} = \cos \chi$ exact, and the operational rates extracted from the integrated trajectory match the algebraic fractions at every tilt.

Proposition 2.2 (Dictionary with the P28 observable). *For the same geodesics, the cycle-averaged Hopf observable satisfies*

$$\langle v(q_{\text{rel}}) \rangle = n_{\parallel}^2 = \left(\frac{d\tau}{dt} \right)^2 = 1 - v_{\text{op}}^2,$$

which is precisely P28’s axis-resolved identity $v = n_{\parallel}^2 + (1 - n_{\parallel}^2) \cos \eta$ averaged over the cycle. The instantaneous $v(q_{\text{rel}})$ oscillates with η ; the invariant content is the cycle average. This fixes the gauge ambiguity P28’s remark after Thm. 5.2 identified (“the missing degree of freedom is the axis”): with the operational dictionary, the axis component $n_{\parallel}$ is the clock rate, the latitude oscillation is gauge, and the physical velocity is $v_{\text{op}} = \sqrt{1 - \langle v \rangle}$. P28’s boxed lapse theorem is correct as stated once $v(q_{\text{rel}})$ is read through this dictionary; what was a postulate becomes bookkeeping.

Corollary 2.3 (NPI; the null cone is the no-winding cone). *Pure base motion ($n_{\parallel} = 0$) carries zero fiber windings: no clock ticks, $d\tau/dt = 0$, while moving at the front speed $v_{\text{op}} = 1$. Phase fronts are therefore null by construction — the Null Phase Identification of A262 is proven, not assumed. Conversely $v_{\text{op}} = 0$ (pure fiber) gives $d\tau/dt = 1$: rest is pure winding, the I-state direction, consistent with the i_Paper’s reading of self-alignment as pure time.*

Corollary 2.4 (Upgrades). *(a) A262’s metronome theorem is now unconditional within the canon: C- η is the fold clock, full stop — the fold-thread observational programme ($z_1 \approx 17$ –26 seeds coupled to the $w(z)$ step at $z_4 \approx 4$ –6) tests the canon itself. (b) P28 Remark 3_c should be annotated closed-relative-to-canon; the residual freedom is exactly the freedom to reject commitments (i) or (ii), each of which now carries an independent closed observable (A239’s T_{breath} ; A256–A261’s staircase phenomenology). (c) P30’s GON benchmarks acquire a sharper reading: the “velocity” input of its scenarios is v_{op} , and prediction (1) (zero error on the Hopf identity) is the Pythagoras of Theorem 2.1.*

3 Status

P28_3_c / OI-262-1: CLOSED relative to canon. The velocity identification is derived from the breath-time commitment and the fold-space commitment; the standalone postulate is

dissolved. What would reopen it: a demonstration that fiber winding is *not* proper time (contradicting A239/A262) or that the Hopf base is *not* space (contradicting the fold ontology and the A256–A261 phenomenology). The canon’s oldest joint is now braced by its newest results — the deep-read of 2026-06-09 flagged this joint in the morning; the fold thread closed it by night.

Open residue. P28’s TBS 28_2 (the $\sqrt{1 - \kappa}$ factor in the lifted arc-speed, a 0.15% normalization question) is untouched and inherited; it perturbs the front speed’s *value*, not the identities proven here, which are ratios.

Verifier. Lumen/corpus/addenda/verify/verify_P263.py — all checks pass, exit 0.