

Addendum 261: Hopf-Charge Superselection

OI-259-1 closed: the fold-transfer fraction is zero,
and the fold defect thermalizes as Wheel heat

TOE Corpus

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Skill: gap-closure v1.1 — **Outcome:** Case B (negative closure — and a consistency success: the kernel independently delivers the decoupling that observation demands) — **Verifier:** `verify_P261.py` — all PASS

Abstract

A259 reduced the fold-native sourcing question to one unknown: the transfer fraction s from the annihilated Hopf-charged sector into scalar curvature perturbations, with $s = \alpha^6$ flagged as a near-miss candidate carrying an Ω_k prediction. This addendum derives s from the kernel. The answer is $s = 0$ **at all orders**: every sector of the canonical master operator commutes with the Hopf fiber charge $\hat{Q}$ — the radial sectors ($D_{B^4}^2$, $V_{\text{self}}^{\text{can}}$, $\alpha\rho$, $\zeta\mathcal{R}$, $\beta\mathcal{M}$) trivially, Δ_{S^3} as the isometry Casimir, Δ_{S^1} as the fiber generator squared, and γT_{cycle} because the lens $\mathbb{Z}_3$ is a subgroup of $U(1)_{\text{Hopf}}$ (A255, verified numerically) — while A189’s Hopf Regularity Axiom forbids any admissible extension from breaking the symmetry, and A218b’s assembly uniqueness removes the loophole of alternative $\hat{O}$ forms. The fold map itself is the charge-zero projection, exactly equivariant. Fiber charge is therefore superselected: the charged sector cannot leak into scalar perturbations through any canonical channel, in either the sudden or adiabatic limit of fold completion. Its energy deposits in the homogeneous background — the corpus identity for which is *Wheel heat conservation* (`kernel/wheel/wheel.py`). Consequences: the fold-native C-ln a PBH channel is closed negatively; the α^6/Ω_k pinning of A259 is demoted to void (the 2.2σ curvature tension evaporates); the LISA mHz prediction inverts — the canon now predicts *absence* of the spike, so a detection would demonstrate structure beyond the canon; A259’s overproduction exclusion is resolved by the kernel itself, a nontrivial consistency success; and the C- η heavy-seed channel survives unchanged, since it requires only standard perturbations plus the fold epoch. OI-259-1 is closed.

1 The Superselection Theorem

Let $\hat{Q} = -i\partial_\psi$ generate the Hopf $U(1)$ on S^3 , with integer charge q labelling the fiber sectors of A259 Prop. 1.1.

Theorem 1.1 (Hopf-charge superselection of the canonical $\hat{O}$). $[\hat{O}, \hat{Q}] = 0$, *sector by sector*:

- $D_{B^4}^2$, $V_{\text{self}}^{\text{can}}(r)$, $\alpha\rho(r)$, $\zeta\mathcal{R} = \zeta r\partial_r$, $\beta\mathcal{M}$: *purely radial — they act on r and spectral weight, not on ψ ; commutation is immediate.*
- Δ_{S^3} : *the Casimir of the isometry group, which contains $U(1)_{\text{Hopf}}$; a Casimir commutes with every generator.*
- Δ_{S^1} : *equals $-\hat{Q}^2$ up to the fiber normalization; commutes with $\hat{Q}$ identically.*

- γT_{cycle} : the family cycle is the lens $\mathbb{Z}_3$ deck action, and by A255 the lens generator acts as $(z_1, z_2) \mapsto (\epsilon z_1, \epsilon z_2)$ with $\epsilon = e^{2\pi i/3}$ — exactly the Hopf $U(1)$ element at $\psi = 2\pi/3$. Since $U(1)$ is abelian, $[T_{\text{cycle}}, \hat{Q}] = 0$. (Verified numerically; the A255 distinction is respected: this is the lens $\mathbb{Z}_3$, not triality.)

Moreover the conclusion is robust beyond the canonical assembly: A189’s Hopf Regularity Axiom requires every admissible sector vector field to be nowhere-vanishing on S^3 , and A189 Thm. 4.1 shows the unique such direction in the centraliser is $H_{U(1)} = \partial_\psi$ itself; and A218b closes assembly uniqueness, so no alternative admissible $\hat{O}$ exists to evade the theorem. Fiber charge is a superselection charge of the kernel.

Corollary 1.2 ($s = 0$). No canonical operator, at any order of perturbation theory, connects the charged sector to the $q = 0$ scalar survivors. The transfer fraction of A259 Cor. 2.3 is $s = 0$ within the canon.

2 The Fold Map Cannot Rescue It

Proposition 2.1 (Equivariance of the fold). The fold completion $S^3 \rightarrow S^2$ acts on modes as the Hopf pushforward — the charge-zero projection P_0 . The Hopf base point $(2z_1\bar{z}_2, |z_1|^2 - |z_2|^2)$ is invariant under the $U(1)$ action (verified numerically to 10^{-12} over random states), so P_0 is exactly equivariant: the fold itself conserves the charge it annihilates — it removes the charged sector’s support, not its charge.

In the sudden limit, the post-fold state is $P_0|\text{pre}\rangle$; the charged components have no image in $L^2(S^2)$ and their energy enters the homogeneous trace. In the adiabatic limit, charged energies $\sim q^2/r_{\text{fiber}}^2$ blueshift as the fiber collapses, and the adiabatic invariant keeps them in charged eigenstates until they exit the spectrum — again into the background. In both limits the deposit is homogeneous: **zero scalar transfer**.

Remark 2.2 (The corpus identity for the deposit: Wheel heat). The kernel already owns the bookkeeping object for “ambiguity/energy that is conserved but not attached to any face”: Wheel heat (`kernel/wheel/wheel.py` line 30: “Heat conservation: total ambiguity is conserved across Fork/Weld pairs”; tested in `kernel/wheel/tests/test_wheel.py`, `TestHeatConservation`). The fold is a Weld-sector event ($\text{Weld} \leftrightarrow \Delta_{S^3}$, Pin 5); the annihilated sector’s energy is Wheel heat. The fold defect thermalizes; it does not gravitate as structure.

Remark 2.3 (NEW THEORY boundary). A nonzero s would require a non-equivariant correction to the fold map — a preferred angular direction during completion. The canon not only lacks such structure; HRA exists precisely to exclude fixed-point-bearing angular directions from the assembly. Any future mechanism delivering $s \neq 0$ is therefore NEW THEORY by construction, and must repeal or weaken HRA explicitly.

3 Consequences

Corollary 3.1 (Channel closure and demotions). (i) $\sigma_{\text{fold}} = \sigma_0$ exactly: no enhancement. With $\delta_c/(\sqrt{2}\sigma_0) \approx 6.9 \times 10^3$, $\log_{10} \beta \sim -10^7$: the fold-native $C\text{-ln } a$ PBH channel is **closed negatively**. (ii) The α^6 candidate and its $\Omega_k \approx -3.5 \times 10^{-3}$ pinning (A259 Rem. 3.2) are **demoted to void** — the 2.2σ curvature tension never materializes; A259’s bracket theorems stand. (iii) The 0.45–0.95 mHz SIGW spike (A257 Prop. 3) **inverts**: the canon predicts its absence; a LISA detection of

a narrow feature there would demonstrate structure beyond the canon (a falsifier turned discovery channel). (iv) A259’s overproduction exclusion (Thm. 2.1) demanded that the fold nearly decouple from the scalar sector; Theorem 1.1 shows the kernel guarantees exact decoupling — HRA, adopted in A189 on spectral grounds, independently protects the universe from shredding at fold completions. A consistency success of the assembly. (v) The C- η heavy-seed channel survives untouched: it requires only standard inflationary perturbations plus the fold epoch (A256/A260); its tests (JWST demographics at $z_1 \approx 17\text{--}26$ coupled to the $w(z)$ step at $z_4 \approx 4\text{--}6$) are now the thread’s primary observational programme.

4 Status

OI-259-1: CLOSED (negative for sourcing; positive for consistency). The “one key, four locks” structure resolves as: the key exists and reads $s = 0$ — it locks the PBH door rather than opening it, releases the curvature lock (R_{CURV} unconstrained by this mechanism), and leaves the clock-unit lock to its A260 reduction ($\equiv B^4$ scale, open as before).

Remaining open in this thread. OI-257-2/259-2 (C- η matter-era demographics), OI-260-1 (fold pacing from the Wheel), OI-260-2 (per-fold arc audit, now lower priority with the anchor demoted), and the B^4 -scale problem (P04), unchanged.

Verifier. `Lumen/corpus/addenda/verify/verify_P261.py` — all checks pass, exit 0.