

Addendum 259: Fold-Event Perturbation Sourcing

Hopf-sector bookkeeping of the $S^3 \rightarrow S^2$ fold, the transfer bracket,
and curvature pinning with the α^6 near-miss

TOE Corpus

2026-06-09

Skill: gap-closure v1.1 — **Outcome:** Case B+ (mechanism bracketed, not closed; one selection rule missing; new testable consequence extracted) — **Task:** TASK_030 — **Verifier:** `verify_P259.py` — all PASS

Abstract

A257 left OI-257-1 open: can the $S^3 \rightarrow S^2$ fold completion source the $\sigma \approx 0.05$ one-scale perturbation spike that the fold-native C-ln a channel requires? This addendum answers with an exact mode bookkeeping and a two-sided exclusion. *First*, under the Hopf map the level- ℓ eigenspace of Δ_{S^3} (dimension $(\ell+1)^2$) splits into a fiber-charge-zero sector of dimension $\ell+1$, which descends exactly to the S^2 harmonics, and a charged sector of dimension $\ell(\ell+1)$, which the fold annihilates: at large ℓ essentially the whole spectrum is lost, and its energy must go somewhere. *Second*, the two natural fates bracket the requirement from both sides: level-local equipartition transfer gives $\sigma \sim \sigma_0 \sqrt{\ell_H} \sim 10^5$ — catastrophic overproduction, excluded by the existence of the universe — while charge-selection transfer ($\Delta q = \pm 1$ only) gives $\sigma = \sqrt{3} \sigma_0 \approx 8 \times 10^{-5}$, sterile. The requirement $\sigma \approx 0.06$ sits strictly inside the bracket, so the mechanism is *possible but not forced*: it needs a transfer fraction s with $s \ell_H = (\sigma_{\text{req}}/\sigma_0)^2 = 1.75 \times 10^6$. *Third*, because $\ell_H = k_{J_1} R_{\text{curv}}$, fixing s pins the spatial curvature: the requirement converts a coupling constant into a curvature prediction. At the Planck flatness bound, $s_{\text{req}} = 0.7 - 1.1 \times 10^{-13}$, and $\alpha^6 = 1.51 \times 10^{-13}$ falls inside factor 2 of that range; taking $s = \alpha^6$ *predicts* $R_{\text{curv}} = 7.5 \times 10^4$ Mpc, i.e. $\Omega_k \approx -3.5 \times 10^{-3}$ (closed, as (B^4, S^3) requires) — in $\sim 2\sigma$ tension with Planck+BAO and decisively testable by CMB-S4-class experiments. The identification is flagged as a near-miss candidate (corpus PSLQ-honesty convention), not a theorem. Verdict: OI-257-1 **bracketed**; the missing piece is the transfer selection rule, filed as OI-259-1.

1 Hopf-Sector Decomposition of the Fold

Proposition 1.1 (Mode bookkeeping). *Write $L^2(S^3) = \bigoplus_{\ell} E_{\ell}$ with $\dim E_{\ell} = (\ell+1)^2$ (P18 spectral decomposition, eigenvalues $\ell(\ell+2)$). Under the Hopf fibration $S^1 \rightarrow S^3 \rightarrow S^2$, each E_{ℓ} splits by fiber charge q : the $q = 0$ subspace has dimension $\ell+1$ and descends isometrically to the S^2 harmonic space of the corresponding level, while the $q \neq 0$ complement has dimension*

$$(\ell+1)^2 - (\ell+1) = \ell(\ell+1).$$

The fold completion $S^3 \rightarrow S^2$ (P32 §5) preserves the charge-zero sector and annihilates the charged sector. The annihilated fraction $\ell/(\ell+1) \rightarrow 1$: at horizon scales ($\ell \sim 10^{19}$, below) essentially the entire spectrum is charged.

Remark 1.2 (Formal language). *A254's collapse delta is the right invariant: the fold's component map has kernel rank $\ell(\ell+1)$ per level, the multi-level analogue of the $n-1$ observer-collapse rank. Per A255, the fiber charge here is the lens $\mathbb{Z}_3$ -compatible $U(1)$ charge of the Hopf bundle — it is not the triality C_3 and the bookkeeping never invokes triality.*

2 The Transfer Bracket

Let $\sigma_0 = \sqrt{A_s} = 4.6 \times 10^{-5}$ be the inflationary amplitude carried by every populated mode before the fold, and let $\ell_H = k_{J_1} R_{\text{curv}}$ be the harmonic level at the J_1 horizon scale ($k_{J_1} = 1.5 \times 10^{14} \text{ Mpc}^{-1}$ for the C-ln a survivor at $T = 3 \text{ PeV}$).

Theorem 2.1 (Overproduction exclusion). *Level-local equipartition — the charged sector's perturbation energy deposited democratically into the $\ell+1$ survivors — multiplies the per-mode variance by $1+\ell$. At the horizon level this gives $\sigma = \sigma_0 \sqrt{1+\ell_H} \approx 1.8 \times 10^5 \gg 1$ for any R_{curv} admitted by the flatness bound. The universe would collapse into black holes at the fold scale. Unsuppressed fold-to-scalar transfer is therefore excluded by observation: the fold must very nearly decouple from the scalar sector. This is a genuine observational constraint on fold dynamics — the first one — and it holds independently of whether the fold-native PBH channel is real.*

Theorem 2.2 (Sterility of charge-selection transfer). *If conservation of fiber charge at the fold restricts transfer to adjacent charges ($\Delta q = \pm 1$), the donor count per level is $O(2\ell)$ against $\ell+1$ survivors carrying weight only through the two adjacent sectors: the variance boost is $O(3)$ and $\sigma = \sqrt{3} \sigma_0 \approx 8 \times 10^{-5}$. Then $\beta = \text{erfc}(\delta_c/\sqrt{2}\sigma)$ is zero to all relevant precision ($\delta_c/\sigma \approx 4 \times 10^3$): no population forms.*

Corollary 2.3 (Bracket). *The A257 requirement $\sigma_{\text{req}} \approx 0.06$ lies strictly between the two derivable couplings. The fold-native mechanism is possible but not forced: it requires a transfer fraction*

$$s = \frac{(\sigma_{\text{req}}/\sigma_0)^2}{\ell_H} = \frac{1.75 \times 10^6}{\ell_H},$$

i.e. a selection rule delivering a specific tiny leakage from the charged sector into the scalar survivors. No such rule is currently derivable from the corpus; this is the precise missing piece (OI-259-1).

3 Curvature Pinning and the α^6 Near-Miss

Proposition 3.1 (Curvature pinning). *Since $\ell_H = k_{J_1} R_{\text{curv}}$, Corollary 2.3 converts any fixed transfer fraction s into a prediction for the spatial curvature radius:*

$$R_{\text{curv}} = \frac{(\sigma_{\text{req}}/\sigma_0)^2}{s k_{J_1}}.$$

The mechanism only operates in a closed universe of a definite size — consistent with, and now quantitatively sharpening, the corpus's (B^4, S^3) closed topology. In a flat universe ($R_{\text{curv}} \rightarrow \infty$) any fixed s overproduces; requiring $\sigma = \sigma_{\text{req}}$ at J_1 therefore demands $\Omega_k \neq 0$.

Remark 3.2 (The α^6 near-miss — candidate, not theorem). *At the Planck flatness bound ($|\Omega_k| \leq 2 \times 10^{-3}$ to 7×10^{-4}), the required fraction is $s_{\text{req}} = 1.1 \times 10^{-13}$ to 6.7×10^{-14} . The TOE constant $\alpha^6 = 1.51 \times 10^{-13}$ falls within a factor 1.3–2.2 of this range (α^5 and α^7 miss by 10^2). Taking $s = \alpha^6$ exactly:*

$$R_{\text{curv}} = 7.5 \times 10^4 \text{ Mpc}, \quad \Omega_k \approx -3.5 \times 10^{-3} \text{ (closed)},$$

in $\sim 2.2\sigma$ tension with Planck+BAO ($\Omega_k = 0.0007 \pm 0.0019$) and decisively testable at CMB-S4 / next-generation BAO precision ($\sigma(\Omega_k) \sim 10^{-3}$). Per the corpus PSLQ-honesty convention (A245, A180), this is recorded as a near-miss candidate: the exponent 6 has plausible readings ($2 \cdot \dim S^3$; one factor of α per fold operation in the octonionic count $k_8 = 2k_4$ at $k_4 = 3$ remaining folds — both post-hoc) but no derivation. If OI-259-1 lands on $s = \alpha^6$, the Ω_k prediction becomes the framework’s **fifth kill-shot**, and its sign (closed) is already forced by the arena.

Superseded (2026-06-09, same day; A261). The transfer fraction has since been derived: $s = 0$ by Hopf-charge superselection of the canonical (A261 Thm. 1.1). This remark’s candidate and its Ω_k pinning are void within the canon; the bracket theorems of §?? — and Thm. 2.1’s decoupling demand, which A261 shows the kernel satisfies exactly — stand.

4 Scale Selectivity

The SIGW companion (A257 Prop. 3) requires a narrow spike. Narrowness survives the book-keeping: the deposit lands on modes crossing the horizon within the fold-completion window. An event-like completion (less than a Hubble time in the fold clock — the natural reading of “fold completion” in P32 §5, where k is a discrete counter, not a continuous parameter) restricts the receiving band to $\Delta\ell/\ell_H \lesssim 1$, preserving the narrow-feature character in both surviving clocks. A continuous-process reading would smear the spike and weaken both the PBH formation efficiency and the SIGW visibility; the discrete reading is canonical in the corpus.

5 Verdict and Open Items

OI-257-1 verdict: bracketed (Case B+). The fold-native sourcing mechanism is neither derived nor excluded. What is now proven: (i) the fold annihilates the charged Hopf sector, $\ell(\ell+1)$ modes per level (Prop. 1.1); (ii) unsuppressed transfer is observationally excluded (Thm. 2.1) — the fold nearly decouples from the scalar sector; (iii) charge-selection transfer is sterile (Thm. 2.2); (iv) the viable window requires a specific transfer fraction that pins the curvature radius (Prop. 3.1), with $s = \alpha^6$ as the flagged candidate (Rem. 3.2).

Open items.

1. **OI-259-1 (transfer selection rule).** Derive s from the kernel — what fraction of charged-sector energy couples to the scalar survivors at fold completion. The Wheel’s heat-conservation identity (Fork/Weld cycles; $\text{Weld} \leftrightarrow \Delta_{S^3}$) is the natural corpus anchor for the fold’s energy bookkeeping and has not yet been applied to it.
2. **OI-259-2 (= OI-257-2).** Matter-era analogue for C- η : the annihilated sector’s imprint on the halo mass function at $z \approx 17\text{--}26$.

Verifier. Lumen/corpus/addenda/verify/verify_P259.py — all checks pass, exit 0. Task: Codex/Write/TASK_030 (executed by Cowork directly, 2026-06-09).