

Addendum 257: The Fold-Native Formation Mechanism

Λ -negligibility at J_1 , the amplitude requirement,
and the mandatory LISA-band gravitational-wave companion

TOE Corpus

2026-06-09

Skill: gap-closure v1.1 — **Outcome:** Case A' (A256 assumption (c) corrected by computation; mechanism requirements quantified; new falsifiable companion prediction) — **Verifier:** `verify_P257.py` — all PASS

Abstract

Addendum 256 carried the collapse-burst hypothesis as bridge assumption (c): that the Λ -staircase discontinuities trigger gravitational-collapse bursts. This addendum tests and corrects it. *First*, the Λ -mediated reading is dead: at the J_1 jump the vacuum-energy fraction is $\rho_{\text{DE}}/\rho_{\text{rad}} \approx 2 \times 10^{-74}$ for the C- $\ln a$ survivor (PeV era) and $\rho_{\text{DE}}/\rho_m \approx 2\text{--}8 \times 10^{-4}$ for C- η ($z \approx 17\text{--}26$); a factor-2 jump in a 10^{-74} component moves no collapse threshold. The staircase marks fold completions; it cannot cause formation. If fold-seeding is real, the mechanism is the fold event itself — *fold-native*. *Second*, the mechanism's job is sharply quantified: producing the C- $\ln a$ population at $M \approx 10^{24}$ g requires formation fraction $\beta \approx 1.5 \times 10^{-13} f_{\text{PBH}}$ and perturbation amplitude $\sigma \approx 0.05\text{--}0.06$ at the single J_1 horizon scale — a $\sim 10^3$ boost over the inflationary baseline, nearly independent of f_{PBH} (Gaussian-tail statistics make the amplitude almost binary). *Third*, such a one-scale spike necessarily induces a narrow scalar-induced gravitational-wave background peaking at $f \approx 0.5\text{--}0.9$ mHz — mid-LISA band — with $\Omega_{\text{GW}} \sim \sigma^4$ well above LISA sensitivity for a narrow feature. Because σ barely depends on f_{PBH} , LISA non-detection at that frequency kills the fold-native C- $\ln a$ channel outright, independently of microlensing. All numbers certified by `verify_P257.py`.

1 Λ -Negligibility at J_1

Proposition 1.1 (The staircase is marker, not mechanism). *At the J_1 epoch of either A256 survivor, the dark-energy fraction of the total density is*

$$\left. \frac{\rho_{\text{DE}}}{\rho_{\text{rad}}} \right|_{T=3 \text{ PeV}} = 1.8 \times 10^{-74}, \quad \left. \frac{\rho_{\text{DE}}}{\rho_{\text{rad}}} \right|_{T=6 \text{ PeV}} = 1.1 \times 10^{-75}$$

for C- $\ln a$ (with $\rho_{\text{DE}} = 2\Lambda_0\rho_{\text{crit},0}$ and $\rho_{\text{rad}} = \frac{\pi^2}{30}g_*T^4$, $g_* = 106.75$), and

$$\left. \frac{\rho_{\text{DE}}}{\rho_m} \right|_{z=17} = 7.5 \times 10^{-4}, \quad \left. \frac{\rho_{\text{DE}}}{\rho_m} \right|_{z=26} = 2.2 \times 10^{-4}$$

for C- η . Collapse-threshold shifts scale with the dark-energy fraction; the J_1 discontinuity in Λ therefore has no dynamical effect on formation at either epoch. The Λ staircase is the marker of fold completions, not the formation mechanism.

Remark 1.2 (Scope of the correction). *A256’s map-class results (Endpoint Lemma, z_4 squeeze, grid theorem, survivor signatures) are unaffected: none invoked the mechanism. What changes is the causal story: formation bursts at J_1 , if real, must be sourced by the fold event itself — the topological collapse $S^3 \rightarrow S^2$ acting on sub-horizon perturbations — a kernel-level question filed below as OI-257-1 and adjacent to OI-256-1 (the fold clock).*

2 What the Fold-Native Mechanism Must Deliver

Proposition 2.1 (Amplitude requirement). *For the $C\text{-ln } a$ mass $M = 1.1 \times 10^{24} \text{ g}$, the standard radiation-era abundance relation $\beta \approx 6.4 \times 10^{-9} f_{\text{PBH}} \sqrt{M/M_\odot}$ and Gaussian-peaks statistics $\beta = \text{erfc}(\delta_c/\sqrt{2}\sigma)$ with $\delta_c = 0.45$ give*

$$\begin{aligned} f_{\text{PBH}} = 1 : & \quad \beta = 1.5 \times 10^{-13}, \quad \sigma = 0.061, \\ f_{\text{PBH}} = 10^{-2} : & \quad \beta = 1.5 \times 10^{-15}, \quad \sigma = 0.056, \\ f_{\text{PBH}} = 10^{-4} : & \quad \beta = 1.5 \times 10^{-17}, \quad \sigma = 0.053, \end{aligned}$$

a boost of $\sim 1.1\text{--}1.2 \times 10^3$ over the inflationary baseline $\sigma \approx 5 \times 10^{-5}$, concentrated at the single J_1 horizon scale.

Corollary 2.2 (Near-binarity). *Because $\sigma \propto 1/\sqrt{\ln(1/\beta)}$, the required amplitude moves only $\sim 15\%$ while f_{PBH} moves four orders of magnitude. The fold either delivers $\sigma \approx 0.05$ and produces a population, or it does not and produces essentially nothing. There is no tuning freedom in which the mechanism is real but the population is unobservably small.*

3 The Mandatory Gravitational-Wave Companion

Proposition 3.1 (SIGW companion). *A narrow scalar-power spike at the horizon scale of M induces a stochastic gravitational-wave background peaking at $f_{\text{peak}} \approx 3 \text{ nHz } (M/30M_\odot)^{-1/2}$:*

$$M = 6 \times 10^{23} \text{ g} : f = 0.95 \text{ mHz}; \quad M = 1.1 \times 10^{24} \text{ g} : f = 0.70 \text{ mHz}; \quad M = 2.6 \times 10^{24} \text{ g} : f = 0.45 \text{ mHz}$$

— the entire $C\text{-ln } a$ band sits inside the LISA window ($10^{-4}\text{--}10^{-1} \text{ Hz}$). The amplitude scales as $\Omega_{\text{GW}} \sim A^2$ with spectral amplitude $A = \sigma^2 \approx 2.5\text{--}3.7 \times 10^{-3}$, far above LISA’s narrow-band sensitivity.

Corollary 3.2 (Fourth kill-shot). *Combining Corollary 2.2 with Proposition 3.1: if the fold-native $C\text{-ln } a$ channel produces any PBH population at all, a narrow SIGW peak must be present at 0.45–0.95 mHz. LISA non-detection at that frequency and amplitude falsifies the channel independently of M31 microlensing. Conversely, a LISA detection of a narrow mHz feature with no inflationary explanation would be the framework’s first positive signature.*

Inverted (2026-06-09, same day; A261). *A261 derives $s = 0$ (Hopf-charge superselection): the canon produces no fold-native population and hence predicts the absence of this spike. The frequency computation above stands; its logical role inverts — detection at 0.45–0.95 mHz would demonstrate structure beyond the canon.*

4 Status and Open Items

What this addendum establishes. Assumption (c) of A256 is corrected (Proposition 1.1); the fold-native mechanism’s requirements are quantified (Proposition 2.1); and the framework gains its

sharpest falsifier yet (Corollary 3.2). The dark-matter half of the original fold-seeding picture is now precise: fold-seeding supplies dark matter only if the fold event sources a $\sigma \approx 0.05$ one-scale spike, and that claim is LISA-decidable.

Open items.

1. **OI-257-1 (mechanism derivation).** Derive from the kernel what the $S^3 \rightarrow S^2$ fold completion does to sub-horizon perturbations — specifically whether it can source $\sigma \approx 0.05$ at the horizon scale. Adjacent to OI-256-1; both reduce to the dynamics of the fold event, which the corpus currently treats kinematically.
2. **OI-257-2 (C- η analogue).** The C- η survivor's J_1 is matter-era; its fold-native signature would be a feature in the halo mass function at $z \approx 17$ –26 rather than SIGW. Quantify once 21 cm forecasts firm up.

Verifier. `Lumen/corpus/addenda/verify/verify_P257.py` — all checks pass, exit 0. Derivation script: `Ged/step_outputs/fold-dm-mechanism/s1_s3_quantify.py`. Plan: `Ged/plans/fold-dm-mechanism.js`