

Addendum 256: The Fold-to-Redshift Map

Partial closure of Conjecture 32.2; the Λ staircase,
the z_4 squeeze, and fold-seeded black holes

TOE Corpus

2026-06-09

Skill: gap-closure v1.1 — **Outcome:** Case B+ (map class reduced to two robust candidates; full closure pending one observational discriminator) — **Verifier:** `verify_P256.py` — 47/47 PASS

Abstract

Paper 32 §5 records the fold-to-redshift mapping as Conjecture 32.2, and `32_Addendum_DynamicLambda` derives the fold-progress dark energy law $\Lambda(k) = \Lambda_0 \cdot 4/C(k)$ without dating any fold. This addendum constrains the mapping $z(k)$ far enough to extract falsifiable cosmology. Three results. *First*, the Euler oscillation $\chi(S^n) = 1 + (-1)^n$ makes $\Lambda(k)$ a three-level *staircase* ($+\infty \rightarrow 2\Lambda_0 \rightarrow \Lambda_0$) with exactly two discontinuities — at the S^2 -fold and S^0 -fold completions — so the fold history imprints on cosmology at exactly two epochs, not four. *Second*, an Endpoint Lemma plus a distance-to-last-scattering squeeze confine the S^0 -fold completion to $z_4 \in [4, \sim 10]$: the $2\Lambda_0 \rightarrow \Lambda_0$ step happened *before* dark-energy domination, which explains the clean Λ_0 measured today and decouples the staircase from low-redshift $w_0 w_a$ fits. *Third*, an exhaustive scan of the natural map class — four cosmological clocks $\{a, \eta, t, \ln a\} \times$ two spacing rules — leaves exactly two robust survivors in the testable window $z_4 \in [4, 6]$: equal conformal spacing (C- η), which places a heavy-seed epoch at $z_1 \approx 17$ –26 compatible with JWST overmassive black holes (UHZ1), and equal logarithmic expansion (C- $\ln a$), which predicts a primordial black hole population at $M \approx 0.6$ – 2.6×10^{24} g from a PeV-scale S^2 -fold. A structural by-product: every surviving map uses *equal-fold* spacing; the Θ -cycle ratio fails in every clock — the Θ -cycle is the phase ledger of the fold history, not its pacemaker. All quantitative claims are certified by `verify_P256.py` (47 checks, exit 0).

1 The Λ Staircase

Paper 32 §5 establishes the boundary collapse sequence $B^4 \rightarrow S^3 \rightarrow S^2 \rightarrow S^1 \rightarrow S^0$ with four active boundary folds $k = 1, \dots, 4$, and the addendum `DynamicLambda` fixes

$$\Lambda(k) = \Lambda_0 \cdot \frac{4}{C(k)}, \quad C(k) = \sum_{i=1}^k \chi(\partial M_i), \quad \Lambda_0 = 1 - \frac{\pi^2}{32}. \quad (1)$$

Proposition 1.1 (Staircase structure). *Because $\chi(S^3) = \chi(S^1) = 0$ and $\chi(S^2) = \chi(S^0) = 2$, the cumulative sum is $C = (0, 0, 2, 2, 4)$ for $k = 0, \dots, 4$, and (1) is a three-level staircase:*

$$\Lambda = +\infty \quad (k \leq 1), \quad \Lambda = 2\Lambda_0 \approx 1.383 \quad (k = 2, 3), \quad \Lambda = \Lambda_0 \approx 0.6916 \quad (k = 4),$$

with exactly two discontinuities: at the completion of the S^2 -fold (jump $J_1: +\infty \rightarrow 2\Lambda_0$) and of the S^0 -fold (jump $J_2: 2\Lambda_0 \rightarrow \Lambda_0$).

Proof. Direct evaluation of (1); verifier checks 1–8. $\square$

Remark 1.2 (Two epochs, not four). *The odd-sphere folds (S^3 , S^1) are cosmologically silent: they advance k without changing Λ . Any observable imprint of the fold history is therefore concentrated at J_1 and J_2 . In particular the pre-fold regime $\Lambda = +\infty$ persists through the completion of the S^3 -fold; the natural end-of-inflation marker in this framework is the S^2 -fold completion, not the S^3 -fold.*

Remark 1.3 (Collapse-burst hypothesis). *A discontinuous drop in vacuum energy abruptly lowers the effective collapse threshold for overdensities crossing the horizon near the jump. The working hypothesis of this addendum — the fold-seeding hypothesis — is that gravitational-collapse activity (primordial black holes in the radiation era, heavy seeds in the matter era) clusters at the redshifts of J_1 and J_2 . Since J_2 post-dates structure formation (§2), the single predicted formation epoch is $z(J_1)$.*

Erratum (2026-06-09, same day; full treatment in Addendum 257). *The Λ -mediated reading of this hypothesis is quantitatively untenable: at J_1 the vacuum-energy fraction is $\rho_{\text{DE}}/\rho_{\text{rad}} \sim 10^{-74}$ ($C\text{-ln } a$, PeV era) and $\rho_{\text{DE}}/\rho_m \sim 10^{-4}$ ($C\text{-}\eta$, $z \approx 17\text{--}26$), so the Λ jump cannot shift any collapse threshold. The staircase discontinuity is the marker of the fold completion, not the formation mechanism; the mechanism, if real, is the fold event itself (fold-native). Every map-class result in this addendum (§2–§4) is unaffected — none used the mechanism — but bridge assumption (c) of §5 should be read as restated by A257, which quantifies what the fold-native mechanism must deliver and derives its mandatory gravitational-wave companion signature.*

2 The Endpoint Lemma and the z_4 Squeeze

Write $z_4 = z(J_2)$ and $z_1 = z(J_1)$ for the redshifts of the two jumps. All background quantities below use the flat Λ CDM bridge ($\Omega_m = 0.315$, $\Omega_\Lambda = 0.685$, $h = 0.674$, $\Omega_r = 9.15 \times 10^{-5}$); above z_4 the dark-energy density is doubled per Proposition 1.1.

Lemma 2.1 (Endpoint Lemma). *Any map $z(k)$ that pins the fold-4 completion to the present epoch ($z_4 = 0$) is excluded, independently of how the earlier folds are spaced.*

Proof. With $z_4 = 0$, $\rho_{\text{DE}} = 2\Lambda_0 \rho_{\text{crit},0}$ for all $z > 0$. Then (i) $\Omega_{\text{DE}}(z=0.5) = 0.56$, a dark-energy-dominated state at an epoch where SN Ia Hubble-diagram fits require matter domination at the $\sim 5\%$ level; and (ii) the DESI DR2 Ly α BAO anchor at $z_{\text{eff}} = 2.33$ shifts by $\delta D_H/D_H = +2.7\%$ against a $\sim 1.2\%$ measurement. Both exceed their gates by factors ≥ 2 (verifier checks 13–14). Spacing rules only rescale $z(J_1)$; they cannot move J_2 off $z = 0$. $\square$

Lemma 2.1 kills the endpoint-normalized axis of the candidate space wholesale — including the two superficially attractive maps “ Θ -phase = scale factor” (whose J_1 falls at $z = 13.0$, squarely in the JWST heavy-seed epoch) and “ Θ -phase = log-temperature fraction”. The fold-4 completion must predate the present.

Proposition 2.2 (z_4 squeeze). *The $2\Lambda_0$ plateau shifts the comoving distance to last scattering while leaving the sound horizon unchanged ($z_4 \ll z_*$), hence $\delta\theta_*/\theta_* = -\delta D_*/D_*$. Numerically*

$$\frac{\delta D_*}{D_*}(z_4) = \{-0.35\%, -0.13\%, -0.063\%, -0.033\%, -0.004\%\} \quad \text{at} \quad z_4 = \{2, 3, 4, 5, 10\}.$$

Against the Planck θ_* precision of 0.03% (gate 6×10^{-4} at 2σ , before parameter-degeneracy relaxation) and the Ly α floor $z_4 > 2.4$ of Lemma 2.1(ii), the fold-4 completion is confined to

$$z_4 \in [4, \sim 10]$$

where the upper end marks loss of testability rather than exclusion: for $z_4 \gg 10$ the plateau is dynamically negligible and the staircase becomes observationally indistinguishable from Λ CDM.

Remark 2.3 (Three consequences). (i) We measure a clean Λ_0 today because J_2 predates dark-energy domination. (ii) The staircase is decoupled from the DESI $w_0 w_a$ low-redshift hint — the framework neither relies on it nor is threatened by its resolution. (iii) The step remains falsifiable: projected Stage-II / 21 cm surveys reach $\sim 2\%$ sensitivity on $\Omega_{\text{DE}}(z)$ out to $z \approx 5$, directly probing the lower half of the window.

3 The Candidate Grid

Definition 3.1 (Map class). A fold-to-redshift map is a pair (clock, spacing): a monotone cosmological clock $X \in \{a, \eta, t, \ln a\}$ (scale factor, conformal time, cosmic time, log-expansion from the Planck epoch) and a spacing rule fixing the clock-ratio between fold completions, either equal-fold ($X(J_1)/X(J_2) = 2/4$) or Θ -ratio ($X(J_1)/X(J_2) = \Theta(\frac{1}{2})/\Theta(1) = 0.0713$, the half-cycle phase fraction of $\Theta(x) = 4\pi^3 x^4 + \pi^2 x^3 + \pi x^2$, Paper 28 §7). The endpoint z_4 is free, subject to Proposition 2.2. No object outside the corpus or standard background cosmology enters (the addendum-eligibility condition of DynamicLambda’s Status note).

Theorem 3.2 (Grid verdicts). Over the eight cells of Definition 3.1, scanned across $z_4 \in \{4, 5, 6, 8, 10\}$, exactly two cells are robustly viable in the testable window $z_4 \in [4, 6]$:

Clock	Equal-fold ($r = 1/2$)	Θ -ratio ($r = 0.0713$)
a	tail-only: $z_1 = 2z_4 + 1 \in [9, 13]$, too late	dead: pre-halo ($z_1 \gtrsim 70$)
η	C-η : $z_1 \in [17, 26]$, DCBH window	dead: dark ages ($z_1 \sim 300\text{--}900$)
t	tail-only: $z_1 \in [9, 13]$ for $z_4 \leq 6$	marginal: grazes at $z_4 = 4$ ($z_1 \approx 28$)
$\ln a$	C-$\ln a$: PBH , $M \approx 10^{24} g$	dead: relic channel ($T \gtrsim 10^{15} \text{ GeV}$)

(Verifier checks 22–37; “tail-only” cells become viable only at $z_4 \geq 8$, where the J_2 step is unobservable.)

Corollary 3.3 (Θ is ledger, not pacemaker). Every robust survivor uses equal-fold spacing; the Θ -ratio rule fails in all four clocks (it compresses J_1 toward the initial singularity in fast clocks and into the dark ages in slow ones). The Θ -cycle records phase accumulated across the fold history ($\Theta(1) = \alpha^{-1}$, the closure of Paper 28 §7); it does not time the folds. Fold completions are uniform in their clock; the phase they deposit is not.

4 The Two Survivors

4.1 C- η : heavy seeds at $z_1 \approx 17\text{--}26$

Equal conformal spacing, $\eta(J_1) = \frac{1}{2} \eta(J_2)$. For $z_4 \in [4, 6]$ the S^2 -fold completion lands at $z_1 \in [17, 26]$ — the matter era, where the horizon-collapse bridge is inapplicable and the natural reading

is a *seeding epoch*: the atomic-cooling-halo window in which direct-collapse seeds of 10^4 – $10^6 M_\odot$ form. A $10^4 M_\odot$ seed at $z \approx 20$ growing Eddington-limited reaches $4 \times 10^7 M_\odot$ by $z = 10.1$, matching UHZ1 and the overmassive black-hole-to-stellar-mass ratios JWST reports at $z \gtrsim 9$.

The map is over-determined: one parameter (z_4) fixes both the $w(z)$ step (future Stage-II/21 cm reach) and the seeding epoch $z_1 = z(\eta = \eta(z_4)/2)$ (JWST seed demographics). Either measurement predicts the other; disagreement kills the map.

4.2 C- $\ln a$: a 10^{24} g PBH population from a PeV-scale fold

Equal log-expansion spacing from the Planck epoch, $\ln(a(J_1)/a_{\text{Pl}}) = \frac{1}{2} \ln(a(J_2)/a_{\text{Pl}})$. The logarithm makes z_1 insensitive to z_4 : across the whole window the S^2 -fold completes in the radiation era at $T \approx 3$ – 6 PeV, giving (with $M_{\text{PBH}} = \gamma M_H$, $\gamma = 0.2$)

$$M_{\text{PBH}} \approx 0.6\text{--}2.6 \times 10^{24} \text{ g} \approx 10^{-9} M_\odot,$$

a band ratio under 5 across $z_4 \in [4, 10]$ (checks 29–30, 41–43). The band sits just above the open asteroid-mass window (10^{17} – 10^{23} g): the original Subaru/HSC M31 limits cap such a population at $f_{\text{PBH}} \lesssim \text{few} \times 10^{-2}$, while finite-source revisions weaken those limits by up to three orders of magnitude, leaving the peak at the boundary of the revised open region. The prediction is decided by next-generation high-cadence M31 microlensing at 10^{-10} – $10^{-8} M_\odot$. The map requires reheating above the PeV scale (the observational floor is $\sim$ MeV; low-scale inflation is admissible but becomes a stated assumption).

4.3 Discriminators

The survivors are mutually exclusive in their primary signature (matter-era seeding epoch vs. radiation-era mass peak) but not logically exclusive — both could hold if the fold clock is scale-dependent, which would be a new object and hence a new paper, not this addendum. The clean discriminators: (i) a measured $w(z)$ step at $z_4 \in [4, 6]$ with *no* seed-epoch excess at $z(\eta=\eta(z_4)/2)$ kills C- η ; (ii) exclusion of $f_{\text{PBH}} > 10^{-4}$ at 10^{24} g kills C- $\ln a$ ’s relevance; (iii) a seed-demographics peak at a redshift inconsistent with every $z_4 \in [4, 6]$ under η -halving kills C- η from the other side.

5 Status, Scope, and Open Items

What this addendum closes. Conjecture 32.2 asked for the fold-to-redshift mapping. This addendum does not produce a unique map; it (i) proves the mapping question reduces to two jumps (Proposition 1.1), (ii) confines the late jump to $z_4 \in [4, 10]$ from existing data (Proposition 2.2), and (iii) reduces the admissible map class from eight natural candidates to two robust survivors with disjoint, falsifiable signatures (Theorem 3.2). Conjecture 32.2 is therefore *partially closed*: open at the level of map uniqueness, closed at the level of phenomenological consequence. Per the DynamicLambda Status note, no new mathematical objects were introduced, so this result remains within the Paper 32 addendum lineage.

Bridge assumptions (stated, not derived). (a) Λ CDM background for the $z \leftrightarrow (\eta, t)$ dictionary; (b) $M_{\text{PBH}} = 0.2 M_H$ horizon-collapse relation in the radiation era; (c) the collapse-burst hypothesis itself (jumps in Λ trigger formation bursts) — plausible from the abrupt change in collapse threshold, not derived from the kernel. A TOE-native replacement for (b) and a derivation of (c) from the fold dynamics are the natural next steps.

Open items.

1. **OI-256-1 (map uniqueness).** Derive the fold clock from first principles. Corollary 3.3 (equal-fold spacing forced; Θ is ledger, not pacemaker) is the structural constraint any derivation must respect; the breath period lacks an SI anchor (A239 closes T_{breath} in system units only), so the unit problem is the same one that blocks the B^4 physical scale (Paper 04).
2. **OI-256-2 (Hubble tension).** Unchanged from Paper 32 Conjecture 6.2; nothing here bears on it.
3. **OI-256-3 (growth and ISW): resolved at first order.** Linear-growth integration through the staircase (fixed early normalization, RK4 on the growth ODE; script `growth_isw_check.py`, verifier checks 45–47) gives a σ_8 deficit that is *sub-percent across the entire window*: 0.02–0.7% depending on how finely the integrator resolves the Λ discontinuity (the spread is sampling jitter at the step, not physics; both resolutions tested agree on magnitude $< 1\%$ and on sign). The sign is *suppression* — the S_8 -tension direction, though far too small to bear on that tension. The potential decay rate $\Phi \sim D/a$ deviates by at most 0.3% over $z \in [z_4, 50]$, so the ISW imprint is invisible in TT. Conclusion: growth observables can neither falsify nor detect the staircase at current or near-future precision (lensing amplitude $\sim 2\text{--}3\%$); the map class is decided exclusively by the expansion-history step ($w(z)$ at z_4) and the J_1 formation signatures of §4. A smoothed-step Boltzmann run would remove the jitter but cannot change the sub-percent verdict.

Verifier. `Lumen/corpus/addenda/verify/verify_P256.py` — 47 checks across six sections (fold machinery, Θ -closure at 50 digits, Endpoint Lemma, z_4 squeeze, full grid, survivor invariants); all pass, exit 0. Derivation script: `Ged/step_outputs/fold-pbh-seeding/s2_derivation.py`. Plan: `Ged/plans/fold-pbh-seeding.json` (s1–s4, complete).