

Addendum 254: Omega as Coequalizer in the Observer Graph

Collapse Delta, Lollipop Compactification,
and the Corrected Status of the P27 Multi-Observer Rank

L. F. Vlegels
Independent Researcher
axiomofchoicepuzzle@gmail.com

May 2026

Abstract

P27 defines Ω as the pre-disagreement state with one observer and, later, as the consensus limit where all observers' projections become consistent. TASK 018 attempted to extend P27's two-observer Time-from-Disagreement theorem to n observers by the graph identity

$$\binom{n}{2} - \beta_1(K_n) = n - 1,$$

but that synthesis was quarantined because it promoted a provisional calculation to Case A' closure and because the coefficient-module question for the P34 associativity defect remained open. TASK 014 independently found an $n - 1$ region-count jump when n bounded lollipop stem-tips are identified to the single point at infinity in the one-point compactification.

This addendum gives the first-pass correction. The shared invariant is not

$$H_1(K_n^\Omega, \Omega; \mathbb{T}),$$

whose rank is $\binom{n}{2}$, but the *collapse delta*

$$\delta_\Omega(A, X) := \text{rank Ker}(H_0(A; \mathbb{T}) \rightarrow H_0(X; \mathbb{T})),$$

where $A = \{O_1, \dots, O_n\}$ is the set of free observer endpoints or lollipop stem-tips in a connected ambient graph X . Since $H_0(A; \mathbb{T}) \cong \mathbb{T}^n$ and $H_0(X; \mathbb{T}) \cong \mathbb{T}$, the component map has matrix $[1 \ 1 \ \dots \ 1]$ and kernel rank $n - 1$. This proves the lollipop $n - 1$ jump and supplies a cleaner structural interpretation of the P27 rank arithmetic.

The result is partial. The OEIS/TASK 018 quantity $D(n) = \beta_1(K_n) - 1$ is not ordinary graph torsion: graph homology over $\mathbb{Z}$ is free, so the $\mathbb{Z}$ and $\mathbb{R}$ ranks agree. If $D(n)$ has a TOE-native homological explanation, it requires the non-trivial P34/Kleisli coefficient module left open in TASK 018. Status: **Case B+** for TASK 018; rank-level value confirmed, torsion/coefficient level open.

Contents

1	Context	3
1.1	P27: Omega is a state, not an ordinary peer vertex	3
1.2	The three target appearances	3
2	The Candidate Omega Graphs	3
3	Collapse Delta	4
4	Lollipop Compactification	5
5	Observer Rank and the Correction to TASK 018	6
5.1	TASK 018 status	7
6	The $D(n)$ Gap	7
7	Open Problems	8
8	Conclusion	8

1 Context

1.1 P27: Omega is a state, not an ordinary peer vertex

P27's relevant claims are twofold. First, before disagreement there is one observer:

At Ω —the pre-Bang state—there is one observer. Self-coherence is trivial.

Second, in the Great Attractor passage, P27 identifies Ω as the consensus state:

The attractor Ω is not a place but a state: the state where all observers' projections become consistent.

These two statements make Ω unlike an ordinary additional observer. Adding Ω as a peer vertex would form K_{n+1} , but the lollipop analogy points instead to a coequalizer or collapse operation: many free endpoints become one shared reference.

1.2 The three target appearances

The first-pass research problem was to compare three similar-looking counts:

- (i) TASK 014: identifying n bounded lollipop stem-tips to one point at infinity increases the face count by $n - 1$ in connected arrangements.
- (ii) TASK 018 N4: the rank arithmetic $\binom{n}{2} - \beta_1(K_n) = n - 1$ matches P27's $n = 2$ base case.
- (iii) TASK 018 N5: the OEIS/lollipop surplus candidate $D(n) = \beta_1(K_n) - 1$ appears at $n = 4$ as $D(4) = 2$.

The bridge-first discipline requires these to be related by explicit maps, not identified by resemblance.

2 The Candidate Omega Graphs

Definition 2.1 (Reference observer graph). For $n \geq 2$, let K_n be the complete graph on observer vertices $O_1, \dots, O_n$. It has

$$|V(K_n)| = n, \quad |E(K_n)| = \binom{n}{2}, \quad \beta_1(K_n) = \binom{n}{2} - n + 1.$$

Definition 2.2 (Graph cone 1-skeleton). Let $\text{Cone}_1(K_n)$ be the graph obtained by adding one vertex Ω and one spoke edge ΩO_i for each observer vertex. This is only the 1-skeleton of a cone, not the full topological cone with filled 2-cells.

Definition 2.3 (Omega quotient graph). Let K_n^Ω be the quotient graph obtained from K_n by identifying all observer vertices:

$$O_1 = \dots = O_n = \Omega.$$

The original edges of K_n become loops based at Ω .

Proposition 2.4 (First Betti numbers of the two Omega graph operations). *For $n \geq 2$,*

$$\beta_1(\text{Cone}_1(K_n)) = \binom{n}{2} \quad \text{and} \quad \beta_1(K_n^\Omega) = \binom{n}{2}.$$

The two graph operations are distinct, but they do not differ in first Betti number.

Proof. For $\text{Cone}_1(K_n)$,

$$|V| = n + 1, \quad |E| = \binom{n}{2} + n, \quad C = 1.$$

Thus

$$\beta_1 = |E| - |V| + C = \binom{n}{2} + n - (n + 1) + 1 = \binom{n}{2}.$$

For K_n^Ω ,

$$|V| = 1, \quad |E| = \binom{n}{2}, \quad C = 1,$$

where loop edges count as edges. Hence

$$\beta_1 = \binom{n}{2} - 1 + 1 = \binom{n}{2}.$$

□

n	$\beta_1(K_n)$	$\binom{n}{2} - \beta_1(K_n)$	$\beta_1(\text{Cone}_1(K_n))$	$\beta_1(K_n^\Omega)$
2	0	1	1	1
3	1	2	3	3
4	3	3	6	6
5	6	4	10	10

Table 1: TASK 019 W1 exact enumeration.

Remark 2.5 (Negative result). Neither $\text{Cone}_1(K_n)$ nor K_n^Ω produces $n - 1$ through $\binom{n}{2} - \beta_1$. Both produce zero by that expression. The value $n - 1$ is therefore not a first-Betti invariant of the Omega graph operation.

3 Collapse Delta

Definition 3.1 (Collapse delta). Let X be a connected graph and let $A = \{O_1, \dots, O_n\}$ be a set of n distinguished vertices. Over a field $\mathbb{F}$, define the Omega collapse delta by

$$\delta_\Omega(A, X) := \text{rank Ker}(i_* : H_0(A; \mathbb{F}) \rightarrow H_0(X; \mathbb{F})),$$

where $i : A \hookrightarrow X$ is inclusion.

Theorem 3.2 (Omega collapse delta). *If X is connected and A has n vertices, then*

$$\delta_\Omega(A, X) = n - 1.$$

Proof. Since A is a discrete set of n points,

$$H_0(A; \mathbb{T}) \cong \mathbb{T}^n.$$

Since X is connected,

$$H_0(X; \mathbb{T}) \cong \mathbb{T}.$$

The inclusion-induced map sends every endpoint component to the unique component of X . In the natural bases, its matrix is

$$i_* = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}.$$

This map has rank 1. By rank-nullity,

$$\text{rank Ker}(i_*) = n - 1.$$

□

Corollary 3.3 (Explicit small cases). *For $n = 2, 3, 4, 5$, the collapse matrices and kernel ranks are:*

n	i_*	$\text{rank Ker}(i_*)$
2	$\begin{bmatrix} 1 & 1 \end{bmatrix}$	1
3	$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$	2
4	$\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$	3
5	$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix}$	4

4 Lollipop Compactification

Proposition 4.1 (Tip-identification face jump). *Let G be a connected bounded lollipop arrangement graph in the plane with n free stem-tip vertices. Let G^Ω be obtained by identifying those n tip vertices to a single point at infinity. If no edges are removed or added, then the number of faces changes by*

$$F(G^\Omega) - F(G) = n - 1.$$

Proof. Euler's formula for a planar graph with C connected components is

$$F = E - V + 1 + C.$$

For connected lollipop witnesses, identifying the n tips to one point changes

$$\Delta V = -(n - 1), \quad \Delta E = 0, \quad \Delta C = 0.$$

Therefore

$$\Delta F = \Delta E - \Delta V + \Delta C = n - 1.$$

□

Remark 4.2. This is the precise sense in which the lollipop $n-1$ count is Omega-relative. The operation is not “add one more vertex”; it is “coequalize n free vertices to one shared reference.” The invariant that sees the operation is the collapse delta of Theorem 3.2.

n	V	E	C	F	V^Ω	E^Ω	C^Ω	$F^\Omega - F$
2	11	18	1	9	10	18	1	1
3	18	30	1	14	16	30	1	2
4	31	54	1	25	28	54	1	3

Table 2: TASK 019 W2 same-configuration lollipop witnesses imported from TASK 014.

5 Observer Rank and the Correction to TASK 018

Proposition 5.1 (Rank arithmetic on the reference graph). *For the complete observer graph K_n ,*

$$\binom{n}{2} - \beta_1(K_n) = n - 1.$$

Proof. Since K_n is connected,

$$\beta_1(K_n) = |E| - |V| + 1 = \binom{n}{2} - n + 1.$$

Therefore

$$\binom{n}{2} - \beta_1(K_n) = \binom{n}{2} - \left(\binom{n}{2} - n + 1 \right) = n - 1.$$

□

Corollary 5.2 (Collapse-delta reading of the rank value). *The TASK 018 rank value agrees with the Omega collapse delta:*

$$\binom{n}{2} - \beta_1(K_n) = \delta_\Omega(\{O_1, \dots, O_n\}, K_n) = n - 1.$$

Remark 5.3 (Status of the physical interpretation). The equality is structural. The physical interpretation still requires a typed bridge from P27 observer disagreement to the distinguished endpoint set $A = \{O_1, \dots, O_n\}$. P27 supplies the motivation: at one observer there is no disagreement, at two observers communication creates time, and Ω is the consensus state. This addendum does not by itself derive the full P27-to-P34 Kleisli morphism identification left open in TASK 018.

Open Problem 5.4 (Observer-category to collapse-map functor). Construct a functor

$$F : \mathcal{Obs} \rightarrow \mathcal{Collapse}$$

from the observer category — with observers as objects and pairwise disagreements as morphisms — to the category of collapse maps $(A \rightarrow \{\Omega\})$, such that F is natural in n and such that the rank of the induced collapse delta recovers the P27 reconciliation rank. This functor would upgrade Corollary 5.2 from a numerical coincidence to a sourced P27 bridge. Until it is derived, the two $n - 1$ counts are identified only structurally, not physically.

Proposition 5.5 (The rejected relative-H1 theorem). *For the quotient graph K_n^Ω ,*

$$\text{rank} H_1(K_n^\Omega, \Omega; \mathbb{7}) = \binom{n}{2}.$$

In particular, this rank is not $n - 1$ except accidentally at no value $n \geq 2$.

Proof. K_n^Ω has one vertex, namely Ω , and $\binom{n}{2}$ loop edges. In the relative pair $(K_n^\Omega, \{\Omega\})$,

$$C_1(K_n^\Omega, \Omega; \mathbb{T}) \cong \mathbb{T}^{\binom{n}{2}}, \quad C_0(K_n^\Omega, \Omega; \mathbb{T}) = 0.$$

The boundary map is therefore the zero map into zero, and all loop-edge generators survive in H_1 . $\square$

Remark 5.6 (Graph cone 1-skeleton). For the graph cone 1-skeleton, the relative chain group has $\binom{n}{2} + n$ one-cells and n relative zero-cells. The n spoke columns have full rank, so the relative H_1 rank is again

$$\binom{n}{2} + n - n = \binom{n}{2}.$$

Thus neither Omega graph model supports the original relative-H1 theorem.

5.1 TASK 018 status

TASK 018 should be revised to:

Case B+: rank-level value confirmed; coefficient level open.

The numerical value $n - 1$ is confirmed by a cleaner mechanism, but the earlier Case A' synthesis remains over-strong because it did not close the P34 coefficient-module or P27/P34 identification gaps.

6 The D(n) Gap

Proposition 6.1 (No ordinary graph torsion). *For every finite graph G , $H_1(G; \mathbb{Z})$ is a free abelian group. Therefore ordinary graph homology cannot produce a $\mathbb{Z}$ -versus- $\mathbb{R}$ torsion gap.*

Proof. A graph is a one-dimensional CW-complex. Its cellular chain complex is

$$0 \rightarrow C_1(G; \mathbb{Z}) \xrightarrow{\partial} C_0(G; \mathbb{Z}) \rightarrow 0,$$

where both chain groups are free abelian. Since there is no C_2 , one has $H_1(G; \mathbb{Z}) = \text{Ker}(\partial)$, a subgroup of a free abelian group. Subgroups of free abelian groups are free abelian. Hence no torsion subgroup appears. $\square$

Remark 6.2 (Status of $D(n) = \beta_1(K_n) - 1$). The first-pass verdict assigns $D(n)$ to a planar outer-face/cycle obstruction or to a future non-trivial coefficient-system calculation. It is not explained by the Omega collapse delta, by the beta1 difference between cone and quotient, or by ordinary $\mathbb{Z}$ versus $\mathbb{R}$ graph homology. A TOE-native homological explanation would have to specify the P34 associativity-defect module and the action of $\pi_1(K_n)$ on that module.

7 Open Problems

Open Problem 7.1 (P34 coefficient module for $D(n)$). Define the coefficient module A for the P34 associativity defect, including the target group, the cocycle condition, and the action of $\pi_1(K_n)$. Only then can one test whether $D(n) = \beta_1(K_n) - 1$ is a TOE-native coefficient-module phenomenon rather than a generic planar outer-face count.

Open Problem 7.2 (Functorial observer bridge). Construct a functor from the observer category, with observers as objects and disagreements/reconciliations as morphisms, to the category of collapse maps

$$A \rightarrow \{\Omega\}.$$

This would upgrade the collapse-delta interpretation from structural analogy to a sourced P27/P34 bridge.

Open Problem 7.3 (P27 textual revision). Determine whether P27’s Time-from-Disagreement theorem should include a remark distinguishing pairwise communication from the Omega collapse delta. The theorem statement need not change, but the $n > 2$ extension should not be cited through TASK 018’s quarantined Case A’ language.

8 Conclusion

The first-pass computation supports a partial unification. The lollipop compactification jump and the observer-rank value share a precise invariant:

$$\text{rank Ker}(H_0(\{O_1, \dots, O_n\}) \rightarrow H_0(X)) = n - 1.$$

This is the Omega collapse delta. It is a coequalizer-style operation on components, not an additional peer observer and not a relative H_1 theorem.

The $D(n) = \beta_1(K_n) - 1$ phenomenon remains separate. It may yet connect to the TOE through a P34 coefficient module, but ordinary graph homology does not provide that bridge. The correct status is therefore Case B+: the rank-level value is confirmed, while the torsion/coefficient layer remains open.

References

- [1] L. F. Vlegels, *The Second Observer: Why Existence Requires Witness, Why Witness Requires Two, and How Intersubjectivity Generates Geometry*, TOE Corpus, Paper 27 (`27_Paper_SecondObserver.tex`), lines 362 and 386 cited for the two Ω passages.
- [2] L. F. Vlegels, *The Kleisli Monad over S^3 : Categorical Composition of Observer States*, TOE Corpus, Paper 34.
- [3] Codex, *TASK_014 lollipop-bounded-arrangement report and data*, `Codex/Read/TASK_014_lollipop-bounded-arrangement_report.md`, 2026.
- [4] Codex, *TASK_018 P27 multi-observer reconciliation report and staged draft*, `Codex/Read/TASK_018_p27-multi-observer-reconciliation_report.md`, 2026.

- [5] Codex, *TASK_019 Omega-relative observer graph W1-W4 artifacts*, Codex/Read/TASK_019_W1_data.json, TASK_019_W2_data.json, TASK_019_W3_collapse_delta.md, TASK_019_W4_verdict.md, 2026.