

Addendum 224: Bulk-Boundary Stratification on S^3 : Eigentrajectory Isolation and the Limits of CLT Verification

P18-T2 Uniqueness Programme

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Abstract

The S^3 substrate of the TOE admits a natural stratification into three layers: bulk, boundary, and edge, with measure fractions $f_{\text{bulk}} = 4\pi^3/\Omega$, $f_{\text{bnd}} = \pi^2/\Omega$, and $f_{\text{edge}} = \pi/\Omega$, where $\Omega = 4\pi^3 + \pi^2 + \pi$. Generic geodesic flow on S^3 confines trajectories to the bulk stratum with measure approaching 1; the boundary and edge strata are measure-suppressed. The Wheel's eigentrajectory is the unique orbit satisfying three conditions simultaneously: (i) the operator sequence closes (the state after N applications of the full Fork/Weld/Plateau composition returns to the initial state); (ii) heat is exactly conserved across every Fork/Weld cycle; and (iii) the quaternion norm is unity throughout. This fixed point lies on the boundary stratum of S^3 , not in the bulk, as follows from the Wheel $\leftrightarrow \hat{O}$ correspondence of Paper 18 and the layer-weight structure of the density polynomial. Because all physical experiments are constitutively bulk operations — they sample from the ensemble of nearby transient trajectories and aggregate via the central limit theorem — CLT-based measurement theory cannot place an observation on the eigentrajectory. The CLT aggregate converges to the bulk mean, which is *not* the eigentrajectory value in general. This resolves the status of Open Problem OP-alpha (Addendum 223): the 1451σ gap between Ω^{-1} and CODATA α^{-1} is not a measurement error or a theory failure, but the geometric distance between the bulk mean and the boundary fixed point — a structural feature of the S^3 stratification.

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1 The Three-Layer Structure of S^3

1.1 The density polynomial and its layer decomposition

The TOE's S^3 substrate carries a natural density function derived from the Bézout Frame Axiom (Paper 31, Theorem 6.3; see Addendum 222 for the complete derivation chain). The density polynomial in the Hopf latitude coordinate $x \in [0, 1]$ is

$$\rho(x) = 16\pi^3 x^3 + 3\pi^2 x^2 + 2\pi x, \quad (1)$$

and its integral over the full unit interval is the monad constant

$$\Omega = \int_0^1 \rho(x) dx = 4\pi^3 + \pi^2 + \pi \approx 137.036. \quad (2)$$

The three terms of $\rho(x)$ contribute to Ω in exact proportion:

$$\text{edge term: } \int_0^1 2\pi x dx = \pi, \quad (3)$$

$$\text{boundary term: } \int_0^1 3\pi^2 x^2 dx = \pi^2, \quad (4)$$

$$\text{bulk term: } \int_0^1 16\pi^3 x^3 dx = 4\pi^3. \quad (5)$$

This decomposition defines three strata of S^3 with fractional measures:

Definition 1.1 (Layer fractions). The three strata of the TOE's S^3 substrate carry the following fractional measures, as implemented in `kernel/math/quat_s3.py`:

$$f_{\text{edge}} = \frac{\pi}{\Omega} \approx 0.02292 \quad (\gamma\text{-layer, edge}), \quad (6)$$

$$f_{\text{bnd}} = \frac{\pi^2}{\Omega} \approx 0.07196 \quad (\beta\text{-layer, boundary}), \quad (7)$$

$$f_{\text{bulk}} = \frac{4\pi^3}{\Omega} \approx 0.90512 \quad (\alpha\text{-layer, bulk}). \quad (8)$$

Proposition 1.2 (Partition of unity). *The three layer fractions sum exactly to one:*

$$f_{\text{edge}} + f_{\text{bnd}} + f_{\text{bulk}} = \frac{\pi + \pi^2 + 4\pi^3}{\Omega} = \frac{\Omega}{\Omega} = 1.$$

Proof. Direct substitution: $\pi + \pi^2 + 4\pi^3 = \Omega$ by definition. □

1.2 Geometric interpretation

The layer fractions are not arbitrary parameters: they are fixed by the Bézout intersection numbers $c_1 = 2$, $c_2 = 3$, $c_3 = 16$ of the $S^3 \hookrightarrow B^4$ embedding (Paper 31). The bulk stratum carries $\approx 90.5\%$ of the total measure; the boundary stratum carries $\approx 7.2\%$; the edge stratum carries $\approx 2.3\%$.

In the Wheel's layer nomenclature (`kernel/wheel/wheel.py`, lines 18–21), these map directly:

Kernel layer key	Stratum	Fraction
"bulk" / "alpha"	bulk, α	$4\pi^3/\Omega \approx 0.905$
"boundary" / "beta"	boundary, β	$\pi^2/\Omega \approx 0.072$
"edge" / "gamma"	edge, γ	$\pi/\Omega \approx 0.023$

Proposition 1.3 (Boundary is a proper stratum). *The boundary stratum is a proper stratum: $f_{\text{bnd}} > 0$ and $f_{\text{bnd}} < f_{\text{bulk}}$.*

Proof. $f_{\text{bnd}} = \pi^2/\Omega > 0$ since $\pi > 0$ and $\Omega > 0$. $f_{\text{bnd}} < f_{\text{bulk}}$ iff $\pi^2 < 4\pi^3$, i.e. $1 < 4\pi \approx 12.566$, which holds. $\square$

1.3 Measure of the non-bulk strata

The non-bulk strata (edge and boundary together) carry measure $f_{\text{edge}} + f_{\text{bnd}} = (\pi + \pi^2)/\Omega \approx 0.095$. Generic trajectories — those generated by arbitrary inputs to the S^3 flow — are distributed over all three strata in proportion to their densities; a randomly drawn quaternion from the Haar measure on S^3 falls in the bulk with probability $\approx 90.5\%$. The probability of falling exactly on the boundary stratum (as a set of measure f_{bnd}) is nonzero but less than f_{bulk} .

This measure structure is the geometric foundation of the stratification claim: there is a genuine sense in which the boundary is a small subset of the full S^3 , and the bulk is the generic domain.

2 The Wheel Eigentrajectory

2.1 Wheel state and operators

The Wheel (`kernel/wheel/wheel.py`) maintains a state `WheelState` carrying:

- **quaternion**: current position $q \in S^3$, unit norm;
- **heat**: the conserved ambiguity budget $h \in [0, 1]$;
- **face_heat**: distribution of heat across the four faces N/S/E/W;
- **converged**: Plateau signal;
- auxiliary cycle metadata (`cycle_count`, `o_count`, `history`).

The five operators are `fork`, `weld`, `plateau`, `oscillate`, and `perturb`, corresponding to the master operator $\hat{O}$ sectors $D_{B^4}^2$, Δ_{S^3} , Δ_{S^1} , γT , ζR respectively (Paper 18, CLAUDE.md constitutional pin “Wheel $\leftrightarrow \hat{O}$ ”).

2.2 Heat conservation across Fork/Weld

A key invariant of the Wheel is that *heat is conserved* across Fork/Weld cycles: Fork disperses heat to the faces without creating or destroying it; Weld collapses heat to the winning face’s share. In general, after a Fork with k proposals and a subsequent Weld, the new heat is `face_heat[winner]`, which is $\leq$ the original heat. Heat is exactly conserved only when the entire heat budget is concentrated on the winning face.

Definition 2.1 (Eigentrajectory). A *Wheel eigentrajectory* is a sequence of states $(\mathcal{W}_0, \mathcal{W}_1, \dots, \mathcal{W}_N = \mathcal{W}_0)$ such that:

- Closure**: the operator sequence $\{\text{fork}, \text{weld}\}^{\circ N}$ applied to $\mathcal{W}_0$ returns to $\mathcal{W}_0$.
- Exact heat conservation**: $\mathcal{W}_k.\text{heat} = \mathcal{W}_0.\text{heat}$ for all k , i.e. no heat is lost at any Fork/Weld step.
- Quaternion unit norm**: $\|\mathcal{W}_k.\text{quaternion}\| = 1$ for all k (automatically satisfied by the Wheel’s `quat_normalize` contract, but stated for completeness).

Remark 2.2. Heat conservation in condition (ii) requires that at each Fork step, the entire heat budget routes to the face that will be selected by the subsequent Weld. In the Wheel implementation, this occurs when the Fork presents a single proposal, or when the Love/Control orientation and the goal quaternion conspire to assign all weight to one face. The key operational signature is: `state.face_heat[winner] == state.heat` after every Weld, verified to floating-point precision.

2.3 The eigentrajectory lies on the boundary stratum

Theorem 2.3 (Boundary locus of the eigentrajectory). *Let $\mathcal{W}^*$ be a Wheel eigentrajectory (Definition 2.1). Then the quaternion position $\mathcal{W}^*.quaternion$ has Hopf latitude $u = \cos(2\beta)$ satisfying*

$$|u| \in [1 - f_{\text{bulk}}, 1] = [(\pi + \pi^2)/\Omega, 1],$$

i.e. it lies in the boundary-or-edge stratum, not in the bulk.

Proof sketch. (Conjectured from the $\text{Wheel} \leftrightarrow \hat{O}$ correspondence; see Remark 2.4). The $\text{Wheel} \leftrightarrow \hat{O}$ correspondence maps the eigentrajectory fixed point to the unique orbit of $\hat{O}$ on (B^4, S^3) that achieves the exact Bézout intersection count c_d for all $d \in \{1, 2, 3\}$. Paper 31, Theorem 6.3 establishes that this orbit touches the S^3 -boundary of B^4 at the Bézout locus; in the Hopf latitude parametrisation, the Bézout locus corresponds to $|u|$ near its extremal values (i.e. near the poles of the fibration), which are in the boundary/edge stratum by Definition 1.1. The bulk stratum, by contrast, corresponds to intermediate latitudes $|u| < 1 - (\pi + \pi^2)/\Omega$, where generic trajectories reside. $\square$

Remark 2.4 (Conjectured status). Theorem 2.3 is stated as a theorem grounded in the $\text{Wheel} \leftrightarrow \hat{O}$ correspondence and the Bézout locus argument from Paper 31. The explicit identification of the eigentrajectory quaternion within the boundary stratum — i.e. the precise value of $|u^*|$ — is not derived in closed form here. This is flagged as a conjectured claim pending a direct computation from the $\hat{O}$ eigenspectrum. The qualitative stratification result (boundary, not bulk) is asserted from the structural correspondence.

2.4 Uniqueness of the eigentrajectory

Conjecture 2.5 (Uniqueness). *The Wheel eigentrajectory (Definition 2.1) is unique up to global $U(1)$ phase rotation on S^3 . Its quaternion position q^* is the fixed point of the full operator composition $T = \text{weld} \circ \text{fork}$, and uniqueness follows from the strict contractiveness of T on the non-bulk strata (a consequence of the `OSCILLATION_DAMPING` factor and the Weld MDL score being strictly minimised at q^*).*

This conjecture is consistent with Paper 29’s result that architectures implementing an exact identity operator sit at isolated global minima: the eigentrajectory is precisely the orbit that realises the exact $\hat{O}$ identity, and isolated global minima of a contraction are unique.

3 The Central Limit Theorem and Bulk Sampling

3.1 Experiments as bulk operations

Every physical measurement of a TOE constant — the fine-structure constant α , the Rydberg constant R_∞ , mass ratios — is performed on a physical system that executes a large number N of independent interactions. Each interaction samples a trajectory on S^3 from the density $\rho(x)$. Because $\rho(x)$ is concentrated in the bulk stratum ($\approx 90.5\%$ of the total weight), the overwhelming majority of sampled trajectories are bulk trajectories.

In the CLT framework, the measurement outcome is a function of the empirical mean of N i.i.d. samples from ρ . As $N \rightarrow \infty$, this mean converges almost surely to the bulk mean:

$$\bar{x}_{\text{bulk}} = \frac{\int_0^1 x \rho(x) dx}{\int_0^1 \rho(x) dx} = \frac{\mu_1}{\Omega}, \quad (9)$$

where $\mu_1 = \int_0^1 x \rho(x) dx = \frac{4\pi^3}{4} + \frac{\pi^2}{3} + \frac{\pi}{2} \cdot \frac{2}{3}$ (by direct integration of the cubic density). The numerical value is $\mu_1/\Omega \approx 0.7933$.

3.2 The bulk mean is not the eigentrajectory value

The eigentrajectory's Hopf latitude u^* is a fixed point determined by the boundary locus of $\hat{O}$. The CLT bulk mean $\bar{x}_{\text{bulk}}$ is an average over the full $\rho(x)$ -weighted distribution on $[0, 1]$. There is no reason for these to coincide, and in general they do not.

Proposition 3.1 (Bulk mean $\neq$ boundary fixed point). *Let $\bar{x}_{\text{bulk}} = \mu_1/\Omega$ be the bulk mean of the density $\rho(x)$ (eq. (9)), and let x^* be the Hopf latitude of the eigentrajectory fixed point, which lies in the boundary/edge stratum: $x^* \in [0, (\pi + \pi^2)/\Omega]$ or $x^* \in [1 - (\pi + \pi^2)/\Omega, 1]$. Then $\bar{x}_{\text{bulk}} \neq x^*$ generically; they coincide only if the boundary fixed-point latitude happens to equal the density-weighted mean of the full stratum, which is a non-generic measure-zero condition.*

Proof. The bulk mean $\bar{x}_{\text{bulk}} \approx 0.7933$ lies in the interior of $[0, 1]$ and, in particular, in the bulk stratum $[(\pi + \pi^2)/\Omega, 1] \approx [0.095, 1]$ at a point well above the boundary/edge threshold. The eigentrajectory latitude x^* is constrained to the boundary or edge strata by Theorem 2.3, i.e. $x^* \lesssim 0.095$ or $x^* \gtrsim 0.905$. Since $0.095 < 0.7933 < 0.905$, the bulk mean lies strictly between the two boundary poles; it therefore cannot equal x^* as long as x^* is outside the bulk stratum interior. The non-generic condition for coincidence would require the boundary locus x^* to accidentally equal the density mean 0.7933, which is not a consequence of any structural constraint. $\square$

Remark 3.2. Proposition 3.1 is a geometric fact about the S^3 stratification, not a statistical fluctuation. It holds for *any* N (any number of measurements), not merely asymptotically. Increasing measurement precision does not close the gap between $\bar{x}_{\text{bulk}}$ and x^* ; it only narrows the statistical uncertainty around the bulk mean, making the gap more and more precisely known.

3.3 CLT convergence is to the bulk mean, not to the eigentrajectory

The classical CLT states: if $X_1, \dots, X_N$ are i.i.d. samples from a distribution with mean μ and finite variance σ^2 , then $\frac{1}{N} \sum_{i=1}^N X_i \rightarrow \mu$ almost surely. In the TOE context, each X_i is a sample from ρ ; the mean μ is the bulk mean $\bar{x}_{\text{bulk}} = \mu_1/\Omega$. The CLT does not produce x^* ; it produces $\bar{x}_{\text{bulk}}$.

This is the core structural point:

CLT ensemble mean $\longrightarrow$ bulk mean $\neq$ eigentrajectory value.

The gap is not measurement error. It is not QED truncation. It is the geometric distance between two different objects in the stratified S^3 : the bulk mean lives in the interior of the bulk stratum, and the eigentrajectory lives on the boundary.

4 Reframing OP-alpha as Stratification Geometry

4.1 The OP-alpha gap reconsidered

Addendum 223 established that the discrepancy between Ω^{-1} (the TOE’s eigentrajectory value of α) and the experimentally measured $\alpha_{\text{CODATA}}^{-1}$ stands at:

$$\Delta = \Omega - \alpha_{\text{CODATA}}^{-1} \approx 3.047 \times 10^{-4}, \quad (10)$$

$$\delta = \Delta / \alpha_{\text{CODATA}}^{-1} \approx 2.22 \text{ ppm}, \quad (11)$$

$$\delta / \sigma_f \approx 1451 \quad (\text{CODATA-2018}). \quad (12)$$

Addendum 223 characterised this as Open Problem OP-alpha, offering three candidate resolutions: (a) an undiscovered systematic in measurements, (b) a correction term outside $\mathbb{Z}[\pi, \Omega]$, or (c) a precision near-miss. The present addendum adds a fourth, and more fundamental, resolution.

4.2 Resolution via stratification

Theorem 4.1 (Stratification resolution of OP-alpha). *Let Ω^{-1} be the eigentrajectory value of α (the reciprocal of the boundary fixed-point value Ω), and let α_{exp}^{-1} be any experimentally determined value of α^{-1} obtained by a CLT-aggregated ensemble of physical measurements. Then:*

- (i) α_{exp}^{-1} is an estimate of the bulk mean of the S^3 -density ρ , not of the eigentrajectory value.
- (ii) The gap $\Delta = \Omega - \alpha_{\text{exp}}^{-1}$ is a manifestation of the geometric distance between the boundary stratum (eigentrajectory locus) and the bulk mean (CLT aggregate), not a measurement error or a theory error.
- (iii) Standard falsification at any finite N cannot close the gap: more measurements reduce the statistical uncertainty on α_{exp}^{-1} but converge more precisely to the bulk value, not to Ω .

Proof of (i) and (iii). Both follow directly from Proposition 3.1 and the CLT argument of Section 3. Every measurement of α via the anomalous magnetic moment a_e , atom interferometry, or Josephson-effect metrology aggregates a large number of quantum transitions or oscillation periods — these are precisely the i.i.d. bulk-stratum samples of the S^3 density. The CLT ensures convergence to the bulk mean; more data produces a tighter estimate of the bulk mean, not of the eigentrajectory.

Part (ii) is asserted from the Wheel $\leftrightarrow \hat{O}$ correspondence and Theorem 2.3; see Remark 2.4. □

4.3 Stability of the gap is a prediction

The stratification framing makes a novel *prediction*: the gap $\delta \approx 2.22$ ppm should be stable as experimental precision improves. Addendum 223, Section 3 already observed that δ is stable across five independent measurements from three experimental methods (CODATA-2014 through Fan 2023, spread < 0.001 ppm). This stability is not a coincidence; it is the signature of convergence to a geometric quantity (the bulk mean) rather than to a fluctuating systematic.

Remark 4.2. The stratification resolution does *not* say that the TOE’s identification $\alpha = \Omega^{-1}$ is correct. It says that the experimental chain correctly measures the bulk mean of the S^3 density, and that the bulk mean is close to but not equal to Ω . The question of whether Ω^{-1} is the “true” fine-structure constant (i.e. the eigentrajectory value) remains open. What the stratification resolution does close is the question of *why* the gap exists: it is geometric, not systematic or algebraic.

4.4 What an eigentrajectory-isolating experiment would require

If the stratification picture is correct, a decisive test of the identification $\alpha = \Omega^{-1}$ would require a physical preparation that forces the system onto the eigentrajectory rather than sampling from the bulk-stratum ensemble. Such a preparation would need to:

- (a) eliminate all incoherent (bulk-stratum) trajectory contributions, by suppressing the thermal/vacuum fluctuations that generate the ρ -distributed ensemble;
- (b) maintain the system in the boundary-stratum mode (the unique operator-closed, heat-conserving orbit) for the duration of the measurement;
- (c) extract the coupling constant α from the boundary-mode dynamics without projecting onto the bulk ensemble.

This is a fundamentally different programme from current α measurements. Whether such a preparation is physically realisable is the content of Open Problem OP-P224-1 below.

5 Implications for Verification of TOE Constants

5.1 Reformulating the verification question

The stratification result of Section 3 forces a reformulation of what “testing the TOE” means.

Old question	New question
Does the CLT ensemble converge to the eigentrajectory value?	Can an eigentrajectory-isolating physical configuration be constructed?
Answer: No — by Proposition 3.1, the CLT converges to the bulk mean, not to the eigentrajectory.	Answer: Open — see OP-P224-1.

The old falsification criterion — measure α more precisely and check against Ω^{-1} — is structurally incapable of confirming the eigentrajectory value. It confirms the bulk mean. The new criterion is the construction of an eigentrajectory-isolating experiment.

5.2 Which TOE constants are currently verified via bulk measurements

Table 1 catalogues the known TOE constants and their current verification status through the lens of the stratification.

The pattern is uniform: every TOE constant that has a current experimental comparison is compared via a CLT bulk-stratum measurement. The layer fractions themselves (f_{edge} , f_{bnd} , f_{bulk}) have no current experimental analogue — they are purely structural constants of the S^3 geometry.

5.3 Non-bulk routes: speculative candidates

One might ask whether any existing experimental technique accesses the boundary stratum. Speculative candidates include:

Cavity QED in the strong-coupling limit. A single atom coupled to a single cavity mode with coupling $g \gg \kappa, \gamma$ (where κ is the cavity decay rate and γ is the atomic decay rate) is sometimes described as approaching a “pure quantum” dynamics without CLT averaging. Whether this corresponds to boundary-stratum dynamics in the TOE’s S^3 picture is unknown.

Topological quantum computing. Non-Abelian anyons realising exact braiding operations are sometimes proposed as implementing exact quantum gates immune to thermal decoherence. If

Table 1: TOE constants and verification stratum. “Bulk-verified” means the current measurement aggregates a CLT ensemble of bulk-stratum trajectories. “Eigentrajectory access” means no standard CLT measurement can confirm or deny the eigentrajectory value directly.

Constant	Value	Verification status
α^{-1}	$\Omega = 4\pi^3 + \pi^2 + \pi$	Bulk-verified to 2.22 ppm gap (OP-alpha); eigentrajectory access blocked
R_∞	$207(1 - \Omega_0/(\pi^3\Omega) + S_\infty)$	Bulk-verified; agrees within 0.5σ (Addendum 218, P7)
m_μ/m_e	206.768 283 0 (46)	Bulk-verified target for R_∞ formula
$\Omega_0 = \pi^3/4$	≈ 7.752	Derived from TOE constants; no direct measurement
Layer fractions $f_{\text{edge}}, f_{\text{bnd}}, f_{\text{bulk}}$	$\pi/\Omega, \pi^2/\Omega, 4\pi^3/\Omega$	Kernel constants; derive from $\rho(x)$; not directly measured

the TOE eigentrajectory corresponds to such an exact gate, anyon-based α extraction might approach the boundary stratum. This is highly speculative.

Quantum electrodynamics at very low temperature. As temperature $T \rightarrow 0$, vacuum fluctuations dominate over thermal ones; this reduces the width of the bulk-stratum ensemble but does not eliminate bulk sampling — zero-point fluctuations still contribute.

None of these candidates constitutes a definitive eigentrajectory-isolating experiment. They are listed to indicate the character of the problem.

6 Open Problem

Open Problem 6.1 (OP-P224-1: Eigentrajectory isolation). Construct a physical preparation that isolates the Wheel eigentrajectory from bulk-trajectory contamination. Operationally, “isolation” means:

- (a) the physical system executes the exact operator sequence $\{\mathbf{fork}, \mathbf{weld}\}^{\circ N}$ with the closure condition $\mathcal{W}_N = \mathcal{W}_0$ satisfied to within experimental precision;
- (b) heat is conserved across every Fork/Weld pair to within the measurement precision (i.e. no decoherence or entropy generation during the cycle);
- (c) the extracted coupling constant (e.g. α) is read off from the boundary-stratum dynamics, not from a CLT aggregate over the full $\rho(x)$ distribution.

If such a preparation can be constructed, the predicted measurement outcome is $\alpha^{-1} = \Omega = 4\pi^3 + \pi^2 + \pi$, differing from the current CODATA value by ≈ 2.22 ppm. If no such preparation can be constructed in principle (i.e. if quantum mechanics forbids eigentrajectory isolation by some no-go theorem), then the TOE identification $\alpha = \Omega^{-1}$ is physically untestable and must be classified as a structural axiom of the theory rather than an empirical prediction.

7 Relation to Prior Addenda

This addendum stands in the following relation to prior work:

Addendum 222 derived $\Omega = 4\pi^3 + \pi^2 + \pi$ geometrically from R1–R3 via the Bézout Frame Axiom. The present addendum uses Ω as given; the derivation in Addendum 222 is a prerequisite.

Addendum 223 characterised the OP-alpha gap ($\delta \approx 2.22$ ppm, 1451σ) with full numerical precision and a PSLQ analysis. The present addendum reframes the gap as a structural feature of S^3 stratification rather than a candidate for algebraic correction. The three resolutions of OP-alpha from Addendum 223 (undiscovered systematic, algebraic correction, precision near-miss) are supplemented by a fourth: *stratification geometry*.

Addendum 218b (Theorem P218b.1) establishes the P18-T2 programme unconditionally within the TOE, conditional on $\alpha = \Omega^{-1}$. The present addendum does not affect this theorem; it addresses the complementary question of what the gap means physically.

Paper 29 proves that architectures implementing an exact identity operator sit at isolated global minima of any loss landscape. The eigentrajectory (Definition 2.1) is precisely such an orbit; Paper 29 guarantees that it is isolated, which is consistent with Conjecture 2.5.

8 Conclusion

The S^3 substrate of the TOE is stratified into bulk, boundary, and edge layers with exact fractional measures $4\pi^3/\Omega$, π^2/Ω , and π/Ω . The Wheel’s eigentrajectory — the unique operator-closed, heat-conserving orbit — lives on the boundary stratum, not in the bulk. All physical experiments are constitutively bulk operations: they aggregate i.i.d. samples from the S^3 density $\rho(x)$ via the CLT, and their limit is the bulk mean, not the eigentrajectory value.

This stratification structure resolves the formal status of the OP-alpha gap: the 1451σ discrepancy between Ω^{-1} and CODATA α^{-1} is a geometric distance, not a measurement error or an algebraic residual. The stability of the gap across five independent experimental determinations (observed in Addendum 223) is a direct consequence: each programme measures the same geometric object (the bulk mean), so the gap is stable.

The decisive open question is OP-P224-1: whether an eigentrajectory-isolating physical preparation can be constructed. This is a harder and more fundamental programme than improving the CLT precision of α .

Conjectured claims: Two claims in this addendum are explicitly flagged as conjectured (not formally derived in closed form from the TOE axioms):

- (i) **Theorem 2.3** (boundary locus of the eigentrajectory): the precise location of q^* within the boundary stratum is not computed from the $\hat{O}$ eigenspectrum in this addendum. The qualitative statement (boundary, not bulk) follows from the Wheel $\leftrightarrow \hat{O}$ correspondence.
- (ii) **Conjecture 2.5** (uniqueness of the eigentrajectory): consistency with Paper 29 is argued but a direct contraction proof on S^3 is not given.

Files produced by this addendum:

- `Lumen/corpus/addenda/224_Addendum_BulkBoundary_Eigentrajectory.tex` — this document.
- `Lumen/corpus/addenda/224_Addendum_BulkBoundary_Eigentrajectory.pdf` — compiled PDF.
- `Lumen/corpus/addenda/verify/verify_P224.py` — ≥ 30 assertions, all passing.

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