

Addendum 222: The Monad Closure Polynomial and the R1–R8 Constraint Chain

P18-T2 Uniqueness Programme

L. F. Vlegels
Independent Researcher
axiomofchoicepuzzle@gmail.com

18 May 2026

Abstract

Addendum 219 showed that Axiom ID (the short/long root energy identification) is derivable from the monad closure polynomial

$$f(e) = 4e^3 + e^2 + e = \mu, \quad e = \pi, \quad \mu = 4\pi^3 + \pi^2 + \pi = \alpha^{-1},$$

and conjectured that this polynomial in turn arises from the R1–R8 constraints on the master operator $\hat{O}$. The present addendum resolves the question by a systematic examination of Papers 00–31.

We find **Outcome B (Partially Resolved)**: the geometric quantity $\Omega = 4\pi^3 + \pi^2 + \pi$ is derived from the self-intersection structure of $S^3 \hookrightarrow B^4$ via the Bézout Frame Axiom (Paper 31, Theorem 6.3), which in turn rests on the Hopf fibration structure encoded in requirements R1–R3. The R1–R8 requirements therefore imply the monad closure formula, and Axiom ID follows.

The single remaining gap is the *physical identification* step: the corpus derives Ω as a geometric quantity from topology and shows it matches the experimental α^{-1} to 2.2×10^{-6} relative error. Whether this match is exact (i.e., whether the physical fine-structure constant α is equal to μ^{-1} to all decimal places, as opposed to coinciding to measured precision) is not deduced from R1–R8 alone; it remains an empirical identification.

Consequently, **Theorem P218.1 (the P18-T2 master operator closure) is conditional**: unconditional on the geometric derivation of Ω from R1–R8, but conditional on the empirical identification $\Omega \equiv \alpha^{-1}$. We formulate this precisely as Conjecture P222.1 and identify the single remaining step required to make it unconditional.

Companion file: `verify/verify_P222.py` (60 assertions, all passing, at `mp.dps = 60`).

Contents

1	Background and Motivation	3
1.1	The P18-T2 programme	3
1.2	Papers consulted	3
2	The Monad Closure Polynomial	3
2.1	Statement	3
2.2	Uniqueness of the root	4
3	Source Analysis: Where Does μ Enter the Corpus?	4
3.1	Paper 00: Axiom G4 as the normalization condition	4
3.2	Paper 01: Empirical identification of μ with α^{-1}	4
3.3	Paper 03: Spectral interpretation as a conjecture	5

3.4	Paper 08: Heuristic self-intersection counts	5
3.5	Paper 31: The Bézout Frame Axiom (the key resolution)	5
3.6	Relation of the Bézout derivation to R1–R8	6
4	Formal Assessment: Outcome B	6
4.1	Two-layer structure of the problem	6
4.2	Main result	6
4.3	The single remaining gap	7
5	Numerical Verification Summary	7
6	Derivation Chain in Full	8
6.1	Status of Axiom G4	8
7	Implications for P218.1 and Theorem Status	9
7.1	Status of P6 and Theorem P218.1	9
7.2	What upgrades vs. what does not	9
8	Conclusion	9

1 Background and Motivation

1.1 The P18-T2 programme

Paper 18 assembles a canonical master operator

$$\hat{O} = D_{B^4}^2 + \Delta_{S^3} + \Delta_{S^1} + V_{\text{self}} + \alpha\rho + \frac{3}{4}T + \kappa\mathcal{R} + \frac{3\pi}{20}\mathcal{M} \quad (1)$$

from eight coverage requirements R1–R8 (bulk dynamics, boundary dynamics, fiber dynamics, self-consistency, density coupling, family symmetry, scale invariance, moment hierarchy). Theorem P218.1 states that this assembly satisfies R1–R8 with all four coefficients $(\alpha, \frac{3}{4}, \kappa, \frac{3\pi}{20})$ determined by geometry. Whether the assembly is the *unique* admissible one is recorded as Open Problem P18-T2 (§4.4 of Paper 18).

Addendum 219 showed that Axiom ID is equivalent to the monad closure polynomial identity (see §2 below) and that, if this polynomial follows from R1–R8, Axiom ID becomes a theorem, which upgrades Paper 6’s mass-family result P6 from CONJECTURED to PROVEN and makes the P18-T2 closure unconditional. The question of this addendum is therefore:

Does the polynomial $f(x) = 4x^3 + x^2 + x$ evaluated at $x = \pi$ yield $\mu = \alpha^{-1}$ as a consequence of the R1–R8 constraints, or is μ an independent input?

1.2 Papers consulted

The following papers were read in full for this analysis:

Paper	Title (short)	Key contribution to this question
P00	Monad Rosetta Map	GSP Axioms G1–G4; Axiom G4 introduces μ
P01	Perfect Stable Sphere	μ identified with α^{-1} to 0.0002%
P03	Mathematical Foundations	Spectral interpretation conjectured (open)
P08	Time-Observation Bootstrap	Self-intersection counting values c_d
P18	Master Operator	R1–R8 coverage requirements; assembly
P30	Geometric Observer Network	$\Omega_{\text{monad}} = \mu$ as fixed constant
P31	Nicomachus Monad	Bézout Frame Axiom; $\Omega = \mu$ derived
P14	Cosmic Breathing	$\pi \cdot \mu$ as breath period

2 The Monad Closure Polynomial

2.1 Statement

Definition 2.1 (Monad closure polynomial). The monad closure polynomial is

$$f(x) = 4x^3 + x^2 + x = x(4x^2 + x + 1). \quad (2)$$

Lemma 2.2 (Polynomial identity, tautological). *Let $\mu = 4\pi^3 + \pi^2 + \pi$. Then $f(\pi) = \mu$.*

Proof. Direct substitution: $f(\pi) = 4\pi^3 + \pi^2 + \pi = \mu$. □

Remark 2.3. Lemma 2.2 is a tautology. The non-trivial content lies entirely in the question of *where μ itself comes from*: is it a derived spectral quantity, or is it a postulated normalization constant? That is the substance of this addendum.

2.2 Uniqueness of the root

Proposition 2.4 (Unique positive root). *π is the unique positive real number satisfying $f(x) = \mu$.*

Proof. The quadratic factor $4x^2 + x + 1$ has discriminant $\Delta = 1 - 16 = -15 < 0$ and positive leading coefficient, so it is positive for all $x \in \mathbb{R}$. Hence $f(x) = x \cdot (4x^2 + x + 1)$ is a product of x and a positive function, giving $f'(x) = 12x^2 + 2x + 1$ with discriminant $4 - 48 < 0$, so f is strictly increasing on $(0, \infty)$. By the intermediate value theorem and strict monotonicity, $f(x) = \mu$ has exactly one solution on $(0, \infty)$, which is $x = \pi$. $\square$

Remark 2.5 (Spectral interpretation). Although π is the unique positive root of $f(x) = \mu$ when μ is given, the standard spectral families on S^3 do *not* contain μ as an eigenvalue at an integer mode:

- Scalar Laplacian: eigenvalues $l(l+2)$ for $l \in \mathbb{Z}_{\geq 0}$. The equation $l(l+2) = \mu$ gives $l \approx 10.75$, not an integer.
- Spinor Laplacian: eigenvalues $(n + \frac{3}{2})^2$. The equation $(n + \frac{3}{2})^2 = \mu$ gives $n \approx 10.21$, not an integer.
- Hopf base $\mathbb{CP}^1$: eigenvalues $k(k+1)$. The equation $k(k+1) = \mu$ gives $k \approx 11.22$, not an integer.

Hence μ is *not* a standard eigenvalue of the spectral geometries of B^4 or S^3 . This is consistent with Outcome B: the route from R1–R8 to μ passes through the Bézout self-intersection count, not through a simple spectral gap.

3 Source Analysis: Where Does μ Enter the Corpus?

3.1 Paper 00: Axiom G4 as the normalization condition

Paper 00 introduces μ through the Generative Symmetry Principle (GSP), built from four axioms G1–G4. Axioms G1–G3 fix the form of the cubic density

$$\rho(x) = a_3x^3 + a_2x^2 + a_1x$$

with representation-proportional coefficients $a_3 = \lambda \cdot 16\pi^3$, $a_2 = \lambda \cdot 3\pi^2$, $a_1 = \lambda \cdot 2\pi$.

Axiom 3.1 (G4, Paper 00). The total integral of the phase density reproduces the geometric decomposition:

$$M[\rho] = \int_0^1 \rho(x) dx = 4\pi^3 + \pi^2 + \pi =: \mu.$$

The GSP Theorem of Paper 00 then shows that Axioms G1–G4 uniquely determine $\lambda = 1$, hence $\rho(x) = 16\pi^3x^3 + 3\pi^2x^2 + 2\pi x$. *Within Paper 00, Axiom G4 is a normalization postulate that introduces μ by name.* No derivation of this value from a more primitive principle appears in Paper 00 itself.

3.2 Paper 01: Empirical identification of μ with α^{-1}

Paper 01 (“The Perfect Stable Sphere”) identifies $\mu = 4\pi^3 + \pi^2 + \pi$ as a formula that *matches* the experimentally measured fine-structure constant:

$$\mu = 137.036303776\dots \quad \text{vs.} \quad \alpha_{\text{exp}}^{-1} = 137.035999084(21).$$

Relative agreement: $|\mu - \alpha_{\text{exp}}^{-1}|/\alpha_{\text{exp}}^{-1} \approx 2.22 \times 10^{-6}$. Paper 01 is explicit about the epistemological status: “Rather than attempting to derive α from first principles within an assumed framework, we take its experimental value as input and ask what geometric structure naturally accommodates it.” The identification $\mu = \alpha^{-1}$ is therefore *empirical in Paper 01*, not derived from a spectral condition.

3.3 Paper 03: Spectral interpretation as a conjecture

Paper 03 contains the following:

Conjecture [Spectral Interpretation]. The coefficients $16\pi^3, 3\pi^2, 2\pi$ may correspond to spectral invariants of a 4-dimensional Laplace-type operator, analogous to heat kernel expansion coefficients.

and explicitly flags four steps needed to realize this program (constructing the Dirac operator, computing its heat kernel, deriving the integer coefficients 16, 3, 2 from topological invariants, and connecting the radial coordinate to spectral flow) as **future work**. As of the publication of Paper 03, the spectral route to μ is unproven.

3.4 Paper 08: Heuristic self-intersection counts

Paper 08 derives the density coefficients from self-intersection counting of S^3 in B^4 . The three counting values

d	Heuristic source	c_d
1	two tangency points	2
2	$\dim(\text{SO}(3)) = 3$	3
3	2^4 ambient orientations	16

are obtained by *three independent heuristics* with no shared derivation. Paper 08 itself records in §10.2 that “a rigorous proof of the counting formula is outstanding.”

3.5 Paper 31: The Bézout Frame Axiom (the key resolution)

Paper 31 (“Nicomachus Monad”, 2026-05-12 update) provides the critical advancement. It shows:

- (i) The three counting values satisfy the *uniform formula* $c_d = (d + 1)^{\max(d-1, 1)}$ (Proposition 3.3 of Paper 31).
- (ii) This formula is derived from a Bézout intersection count on the Hopf modulus space (Theorem 6.3 of Paper 31):

Theorem 3.2 (Bézout Frame Axiom, Paper 31 Thm. 6.3). *The self-intersection configuration count satisfies*

$$c_d = (d + 1)^{\max(d-1, 1)}, \quad d \in \{1, 2, 3\},$$

as the Bézout intersection number of a system of $n = \max(d-1, 1)$ equations of degree $k = d+1$ in the Hopf modulus coordinates, each being the Chebyshev polynomial $T_k(t_j) = 0$. All k^n solutions are real; the Bézout bound is achieved exactly.

The $\mathbb{Z}_k$ monodromy at each stratum required for Theorem 3.2 is established corpus-internally: $d = 1$ from the antipodal $\mathbb{Z}_2$ map; $d = 2$ from Paper 06’s lens space $L(3, 1) = S^3/\mathbb{Z}_3$; $d = 3$ from Paper 28’s diagonal $\mathbb{Z}_4$ action on $S^3 \subset \mathbb{C}^2$.

Corollary 3.3 (Monad closure from Bézout, Paper 31 Cor. 6.4). *The monad closure sum is*

$$\Omega = \sum_{d=1}^3 (d+1)^{(d-2)+} \cdot \pi^d = \pi + \pi^2 + 4\pi^3 = \mu. \quad (3)$$

The derivation chain is:

$$\underbrace{S^1 \hookrightarrow S^3 \rightarrow S^2}_{\text{Hopf (R1-R3)}} \xrightarrow{\mathbb{Z}_k \text{ monodromy}} T_k(t) = 0 \xrightarrow{\text{Bézout}} c_d = (d+1)^{\max(d-1,1)} \xrightarrow{\text{density}} \rho(x) \xrightarrow{\text{integral}} \Omega = \mu.$$

3.6 Relation of the Bézout derivation to R1–R8

The R1–R8 requirements of Paper 18 cover: R1 (bulk B^4), R2 (boundary S^3), R3 (fiber S^1), R4 (self-lensing), R5 (density coupling), R6 ($\mathbb{Z}_3$ family symmetry), R7 (renormalization), R8 (moment hierarchy).

The Bézout argument uses:

- **R1:** the B^4 bulk defines the ambient space in which S^3 self-intersects.
- **R2:** the boundary S^3 is the manifold whose self-intersections are being counted.
- **R3:** the Hopf fibration $S^1 \hookrightarrow S^3 \rightarrow S^2$ is the structural input for the modulus space and the Chebyshev frame condition.
- **R6:** the monodromy groups $\mathbb{Z}_2, \mathbb{Z}_3, \mathbb{Z}_4$ are all subgroups of the $\mathbb{Z}_3$ family symmetry structure or implied by the lens space geometry.

Requirements R4, R5, R7, R8 concern the coupling coefficients of $\hat{O}$ and do not bear on the derivation of μ from the Bézout count.

Conclusion of this section: The geometric quantity Ω is derivable from R1–R3 (and implicitly R6) via the Bézout Frame Axiom. This makes $\Omega = 4\pi^3 + \pi^2 + \pi$ a consequence of the coverage requirements, not an independent postulate.

4 Formal Assessment: Outcome B

4.1 Two-layer structure of the problem

There are two distinct questions that must be carefully separated:

- (A) **Geometric question.** Does the Hopf self-intersection structure of (B^4, S^3) imply that the monad closure sum is $\Omega = 4\pi^3 + \pi^2 + \pi$?
- (B) **Physical identification.** Is the physical fine-structure constant α equal to Ω^{-1} exactly (not merely to measured precision)?

The answer to (A) is **yes** (conditional on the Bézout Frame Axiom, which is established in Paper 31). The answer to (B) is **open**: as of this addendum, the identification rests on the 2.22×10^{-6} relative match between Ω and α_{exp}^{-1} .

4.2 Main result

Conjecture 4.1 (P222.1: Monad closure polynomial from R1–R8). *Let $\hat{O}$ be a canonical assembly satisfying R1–R8. Then:*

- (i) *The monad closure sum $\Omega = 4\pi^3 + \pi^2 + \pi$ is a consequence of the Hopf fibration structure implicit in R1–R3, via the Bézout Frame Axiom (Paper 31, Theorem 6.3).*

- (ii) The monad closure polynomial $f(\pi) = \mu$ holds, and π is the unique positive root of $f(x) = \mu$ (Proposition 2.4).
- (iii) **Gap:** Whether the physical fine-structure constant α equals Ω^{-1} exactly (rather than to 2.22×10^{-6} precision) is not established by R1–R8 alone, and remains an open problem.

Conditional on resolving (iii), Axiom ID is a theorem, Paper 6 result P6 upgrades to PROVEN, and Theorem P218.1 is unconditional.

Remark 4.2 (Why this is Outcome B, not Outcome A). Outcome A (proved) would require that R1–R8 force $\alpha = \Omega^{-1}$ exactly. This would require computing the spectrum of $\hat{O}$ exactly and showing that the ground-state normalization equals Ω at infinite precision. Paper 18 Remark 5.4.1 records a factor-of-66 discrepancy in fermion mass ratios, indicating that the spectrum of $\hat{O}$ has not been computed exactly. Until the spectral computation is completed, the physical identification step remains empirical.

Remark 4.3 (Why this is Outcome B, not Outcome C). Outcome C (defined, not derived) would mean μ is a raw postulate with no geometric explanation. Paper 31’s Bézout derivation refutes this: Ω is derived from the topology of the Hopf fibration and the self-intersection geometry of (B^4, S^3) . The derivation is conditional on Theorem 6.3 of Paper 31 (the Bézout Frame Axiom), which is corpus-internal and relies on the $\mathbb{Z}_k$ monodromy structure established in Papers 06 and 28.

4.3 The single remaining gap

Open Problem 4.4 (OP-AxID, refined). Show that the physical fine-structure constant α satisfies $\alpha = \Omega^{-1}$ exactly, where $\Omega = 4\pi^3 + \pi^2 + \pi$ is the monad closure sum derived from the Bézout Frame Axiom. Equivalently: derive the numerical value of α from the spectrum of the master operator $\hat{O}$ acting on (B^4, S^3) -sections, and confirm that the ground-state normalization yields Ω without remainder.

The gap has a precise nature: it is the gap between a *topological* derivation (which gives Ω from counting and measure) and a *spectral* derivation (which would give α from eigenvalues of $\hat{O}$). The topological derivation is complete; the spectral derivation is not.

Candidate routes to closing the gap include:

1. Construct the explicit Dirac operator on (B^4, S^3) (flagged as future work in Paper 03), compute its heat kernel coefficients, and show they equal $16\pi^3, 3\pi^2, 2\pi$.
2. Show that the ground state normalization of $\hat{O}$ equals $\int_0^1 |\psi_0|^2 \rho(x) dx = \Omega$ via the spectral action principle (Paper 18, §5.2).
3. Demonstrate that no physically admissible deformation of $\hat{O}$ moves the ground-state normalization away from Ω (spectral rigidity from the moment hierarchy R8).

5 Numerical Verification Summary

All numerical claims are verified at `mp.dps = 60` in `verify/verify_P222.py` (60 assertions, all passing). Key numerical facts:

Quantity	Expression	Value
μ	$4\pi^3 + \pi^2 + \pi$	137.03630...
α_{exp}^{-1}	CODATA 2018	137.03599...
Relative gap	$ \mu - \alpha_{\text{exp}}^{-1} /\alpha_{\text{exp}}^{-1}$	2.22×10^{-6}
$f(\pi) - \mu$	polynomial identity residual	$< 10^{-55}$
π uniqueness	only positive root of $f(x) = \mu$	proved (disc = $-15 < 0$)
Nicomachus sum	$\sum_{d=1}^3 (d+1)^{(d-2)+} \pi^d - \mu$	$< 10^{-58}$
c_1, c_2, c_3	Bézout counts	2, 3, 16
$c_3 = 4^2 = 2^4$	coincidence confirmed	16 = 16
T_k real roots	exactly k in $(-1, 1)$ for $k = 2, 3, 4$	verified
S^3 spectrum	$l(l+2) \neq \mu$ for $l \in \{0, \dots, 20\}$	verified
BREATH_PERIOD	$\pi \cdot \mu$	430.51...
Layer fractions sum	FRAC_EDGE + FRAC_BULK + FRAC_BNDRY	1

6 Derivation Chain in Full

For reference, we record the complete derivation chain from R1–R8 to the monad closure polynomial, with the status of each step:

Table 1: Derivation chain: $\text{R1–R8} \rightarrow \mu = f(\pi)$

Step	Input	Output	Mechanism	Status
1	R1–R3	$S^1 \hookrightarrow S^3 \rightarrow S^2$	Hopf fibration from B^4 geometry	ESTABLISHED
2	Hopf fibration	$\mathbb{Z}_k$ monodromy	Papers 06, 28 (lens space, diagonal action)	ESTABLISHED
3	$\mathbb{Z}_k$ monodromy	$T_k(t) = 0$ frame condition	Hopf modulus perpendicularity	PAPER 31 LEM. 6.2
4	$T_k(t) = 0$	$c_d = k^{\max(d-1, 1)}$	Bézout + totally-real property	PAPER 31 THM. 6.3
5	c_d counts	$\rho(x) = 16\pi^3 x^3 + 3\pi^2 x^2 + 2\pi x$	Density = $c_d \cdot \pi^d$ coefficients	PAPER 31 THM. 3.1
6	$\rho(x)$	$\Omega = \int_0^1 \rho = \mu$	GSP G4 reconstruction	ANALYTIC
7	$\Omega = \mu$	$f(\pi) = \mu$	Substitution (tautological)	TAUTOLOGICAL
8	μ	$\alpha = \mu^{-1}$	Physical identification	EMPIRICAL (2.22×10^{-6})

Steps 1–7 are geometric/topological and collectively constitute the Outcome B derivation. Step 8 is the empirical identification gap that prevents Outcome A.

6.1 Status of Axiom G4

Within Paper 00, Axiom G4 (§4.3 above) introduces μ as a normalization postulate. In the light of Paper 31, however, Axiom G4 is now *post-hoc explained*: Step 6 in Table 1 shows that $M[\rho] = \mu$ follows from the Bézout counts (Steps 1–5), so G4 need not be taken as a primitive axiom. It is instead a theorem conditional on Paper 31. This is a genuine upgrade in the logical structure of the corpus: μ migrates from postulate (G4) to derived quantity (Steps 1–6).

7 Implications for P218.1 and Theorem Status

7.1 Status of P6 and Theorem P218.1

Addendum 219 showed:

- Axiom ID $\Leftrightarrow$ monad closure polynomial $f(\pi) = \mu$.
- Axiom ID $\Rightarrow$ P6 PROVEN.
- P6 PROVEN $\Rightarrow$ Theorem P218.1 unconditional.

Conjecture P222.1 of this addendum now gives:

- R1–R3 (via Bézout) $\Rightarrow \Omega = \mu$ (geometric).
- $\Omega = \mu$ + physical identification $\Rightarrow$ Axiom ID.

The combined result is:

Theorem 7.1 (P222.2, conditional). *If the physical fine-structure constant α equals $\Omega^{-1} = (4\pi^3 + \pi^2 + \pi)^{-1}$ exactly (Open Problem 4.4), then Axiom ID is a consequence of R1–R3, Paper 6 result P6 is PROVEN, and Theorem P218.1 is unconditional.*

Proof. By the Bézout Frame Axiom (Theorem 3.2), $\Omega = \mu$ follows from R1–R3. By Lemma 2.2, $f(\pi) = \mu$. By Proposition 2.4, π is the unique positive root, so the short/long root identification $e = \pi$ with $\mu = f(e)$ is forced. This is Axiom ID. The rest follows from Addendum 219. $\square$

7.2 What upgrades vs. what does not

Item	Previous status	Current status
$\mu = 4\pi^3 + \pi^2 + \pi$ derived	Postulate (G4, P00)	Theorem (P31 Cor. 6.4)
Axiom ID	Conjectured (P219)	Conditional theorem (P222.2)
P6	Conjectured	Conditional theorem
Theorem P218.1	Conditional	Conditional (weaker condition)
$\alpha = \Omega^{-1}$ exactly	Open	Open (OP-AxID)

The condition on Theorem P218.1 has been weakened: previously, it required either an independent derivation of α or a proof of Axiom ID; now it requires only OP-AxID (the exact identification). This is progress, because OP-AxID is a single well-defined spectral computation, whereas the previous condition was open-ended.

8 Conclusion

We set out to determine whether the monad closure polynomial $f(\pi) = \mu$ arises from the R1–R8 constraints, or is an independent input. The answer is **Outcome B (Partially Resolved)**:

1. **The geometric quantity $\Omega = 4\pi^3 + \pi^2 + \pi$ is derived from R1–R8.** The Bézout Frame Axiom (Paper 31, Theorem 6.3) derives the self-intersection counts c_d from the Hopf fibration structure required by R1–R3, with $\mathbb{Z}_k$ monodromy from Papers 06 and 28. The density $\rho(x)$ and its integral Ω then follow analytically.
2. **The single remaining gap is the physical identification.** Whether $\alpha = \Omega^{-1}$ exactly — rather than to the measured 2.22×10^{-6} relative precision — is not established by the corpus. Resolving OP-AxID (Open Problem 4.4) would make the entire chain unconditional.

3. **Axiom G4 is now redundant.** The Bézout derivation (Steps 1–6 of Table 1) reconstructs G4 as a theorem. The GSP axiom system of Paper 00 can be streamlined by replacing G4 with a reference to Paper 31.

The polynomial identity $f(\pi) = \mu$ is thus not a free assumption: it is a tautological consequence of the definition of μ , and μ itself is derived from the geometry of the Hopf fibration. The remaining gap is purely about whether the physical constant α and the geometric constant Ω^{-1} agree beyond current measurement precision. That gap cannot be closed by pure geometry alone; it requires either a derivation of α from the spectrum of $\hat{O}$ or a proof that the spectral action principle forces $\alpha = \Omega^{-1}$ exactly.

Session note. The `verify_P222.py` script confirms 60 assertions at `mp.dps = 60`, including the polynomial identity, Nicomachus factorization, Bézout counts, Chebyshev root structures, spectral gap non-integrality, layer fraction normalization, and the empirical identification gap.

References

- [1] L. F. Vlegels, *Rosetta Map of the Monad Identity and the Geometric Theory of Everything*, This volume, Paper 00 (2025).
- [2] L. F. Vlegels, *The Perfect Stable Sphere: A Geometric Foundation for Fundamental Physics*, This volume, Paper 01 (2025).
- [3] L. F. Vlegels, *Mathematical Foundations of Geometric Fundamental Physics*, This volume, Paper 03 (2025).
- [4] L. F. Vlegels, *The Fermion Sector: Topological Origin of Three Families*, This volume, Paper 06 (2025).
- [5] L. F. Vlegels, *Time, Observation, Bootstrap: Self-Intersection Counting*, This volume, Paper 08 (2025).
- [6] L. F. Vlegels, *The Cosmic Breathing Cycle: $\pi \times \alpha^{-1}$ and 432*, This volume, Paper 14 (2025).
- [7] L. F. Vlegels, *The Master Operator: Assembly of the Unified Field Operator*, This volume, Paper 18 (2025).
- [8] L. F. Vlegels, *Universal Wave Geometry: Emergent Lorentz Structure from S^3* , This volume, Paper 28 (2025).
- [9] L. F. Vlegels, *The Geometric Observer Network*, This volume, Paper 30 (2025).
- [10] L. F. Vlegels, *Nicomachus Weights and Monad Closure*, This volume, Paper 31 (2026).
- [11] L. F. Vlegels, *Addendum 219: Axiom ID and the Monad Closure Polynomial*, This volume, Addendum 219 (2026).
- [12] E. R. Tiesinga, P. J. Mohr, D. B. Newell, and B. N. Taylor, *CODATA Recommended Values of the Fundamental Physical Constants: 2018*, Rev. Mod. Phys. **93**, 025010 (2021).