

Addendum 220

Closure Attempt on Open Problem OP-rSinf-100:
Precision Analysis of the Near-Miss $r \cdot S_\infty \approx 100c$
and its Status under CODATA Uncertainty

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1 Background and context

Papers P216 and P217 of the TOE corpus established that the geometric ratio

$$r = \frac{h E_\mu}{\mu^2} = \frac{3\pi^2}{\mu^2} \approx 1.5767 \times 10^{-3}$$

and the closed-form series limit

$$S_\infty = \frac{h \mu}{E_e^3(\mu^2 + h E_\mu)} = \frac{3\mu}{\pi^3(\mu^2 + 3\pi^2)} \approx 7.0494 \times 10^{-4}$$

(where $\mu = 4\pi^3 + \pi^2 + \pi$, $h = 3$, $E_e = \pi$, $E_\mu = \pi^2$) satisfy the product identity

$$r \cdot S_\infty = \frac{9}{\mu \pi (\mu^2 + 3\pi^2)} \approx 1.11148 \times 10^{-6}. \quad (1)$$

This identity was verified algebraically in P217 and is exact.

Independently, the TOE model predicts the Rydberg-analogue constant

$$\hat{R} = 207 \left(1 - \frac{\Omega_0}{E_e^3 \mu} + S_\infty \right), \quad \Omega_0 = \frac{\pi^3}{4}, \quad (2)$$

giving $\hat{R} \approx 206.76828530$, which is to be compared against the experimental target $R_\infty^{\text{exp}} = 206.7682830$ (CODATA, in the model's working units, with uncertainty $\sigma_T = 4.6 \times 10^{-6}$).

Define the per-prefactor residual

$$c = \frac{\hat{R} - R_\infty^{\text{exp}}}{207} = \frac{\delta_\infty}{207}, \quad \delta_\infty = \hat{R} - 206.7682830. \quad (3)$$

At 60-digit precision (mpmath, `mp.dps=60`):

$$\delta_\infty = 2.30071217 \dots \times 10^{-6}, \quad c = 1.11145515 \dots \times 10^{-8}.$$

The open problem OP-rSinf-100 asks whether the ratio

$$\frac{r \cdot S_\infty}{100 c} \stackrel{?}{=} 1 \quad (4)$$

holds exactly, or whether the near-miss $r \cdot S_\infty \approx 100 c$ is merely coincidental. The name encodes the factor 100.

Remark 1.1. P217 phrased the near-miss as “ $r \cdot S_\infty \cdot 100 \approx c$ ” (a sign reversal and factor-of- 10^4 transcription error carried through from the original notebook). The correct reading, consistent with the formulas therein, is $r \cdot S_\infty \approx 100 c$. The PSLQ null result in P216 was performed on the fractional deviation ε defined below, which is unaffected by this notation issue.

2 Section 1 — Precision pinning of ε and CODATA uncertainty

2.1 Full-precision value of ε

Define

$$\varepsilon := \frac{r \cdot S_\infty}{100 c} - 1. \quad (5)$$

Using `mpmath` at `mp.dps=60` one obtains

$$\boxed{\varepsilon = +2.20367\,25885\,93701\,88910\,63220\,44012\,48147\dots \times 10^{-5}.} \quad (6)$$

This is positive, confirming that $r \cdot S_\infty$ exceeds $100\,c$ by a fractional amount of roughly 2.2×10^{-5} .

For reference the two sides of the near-miss are

$$\begin{aligned} r \cdot S_\infty &= 1.11147964\,7459600611\,2441668291\dots \times 10^{-6}, \\ 100\,c &= 1.11145515\,4627023583\,7816110021\dots \times 10^{-6}, \end{aligned}$$

so the absolute discrepancy is $r \cdot S_\infty - 100\,c \approx 2.449 \times 10^{-11}$.

2.2 The quantity c and its experimental heritage

Since $c = (\hat{R} - R_\infty^{\text{exp}})/207$ and $\hat{R}$ is a pure mathematical quantity (no experimental uncertainty), the only source of uncertainty in c is the CODATA uncertainty in R_∞^{exp} :

$$\sigma_c = \frac{\sigma_T}{207} = \frac{4.6 \times 10^{-6}}{207} \approx 2.222 \times 10^{-8}.$$

The relative uncertainty in c is therefore

$$\frac{\sigma_c}{c} = \frac{2.222 \times 10^{-8}}{1.111 \times 10^{-8}} \approx 1.999.$$

The current CODATA measurement resolves c to only about $\pm 100\%$ relative precision. Equivalently, $\sigma_T \approx 2\delta_\infty$, so the residual δ_∞ is itself uncertain to the same order as its magnitude.

2.3 CODATA propagation to σ_ε

Propagating the uncertainty in TARGET through c to ε :

$$\frac{\partial \varepsilon}{\partial T} = -\frac{r \cdot S_\infty}{100\,c^2} \cdot \frac{\partial c}{\partial T} = \frac{r \cdot S_\infty}{100\,c^2 \cdot 207},$$

giving

$$\sigma_\varepsilon = \frac{r \cdot S_\infty}{100\,c^2 \cdot 207} \sigma_T \approx 1.999. \quad (7)$$

The uncertainty in ε is of order 2 (dimensionless), roughly 10^5 times larger than ε itself.

2.4 Key ratio

$$\frac{\varepsilon}{\sigma_\varepsilon} = \frac{2.2037 \times 10^{-5}}{1.999} \approx 1.1 \times 10^{-5} \ll 1. \quad (8)$$

Conclusion (Section 1): The near-miss $r \cdot S_\infty \approx 100\,c$ cannot be falsified with the current CODATA precision. The deviation ε is buried approximately $10^5 \sigma$ inside the measurement uncertainty, so the present experimental data is entirely consistent with the exact identity $r \cdot S_\infty = 100\,c$. This places OP-rSinf-100 firmly in **Case A**.

3 Section 2 — Extended PSLQ search

P216 ran PSLQ on ε using a 20-element basis of powers of π and μ and obtained a null result. This addendum extends the search to three larger bases.

3.1 Basis 1: powers of π and μ (20 elements)

Elements: $\{\pi^k : k = 1, \dots, 6\} \cup \{\mu^k : k = 1, \dots, 4\} \cup \{\pi^j \mu^k : j, k \geq 1, j + k \leq 5\} \cup \{1, \varepsilon\}$.

Result: PSLQ (maxcoeff = 1000, tol = 10^{-30}) returned the relation

$$\mu^3 - \pi\mu^2 - \pi^2\mu^2 - 4\pi^3\mu^2 = 0,$$

with coefficient vector $(0, \dots, 0, 1, 0, 0, -1, 0, 0, -1, 0, 0, -4, 0, 0)$ and $\max|\text{coeff}| = 4$. This is simply the defining identity $\mu = 4\pi^3 + \pi^2 + \pi$, rewritten as $\mu^3 = (4\pi^3 + \pi^2 + \pi)\mu^2$, which is trivially true. The coefficient of ε in this relation is **zero**: no non-trivial relation involving ε was found.

3.2 Basis 2: algebraic elements

Elements: $\{1, r, S_\infty, rS_\infty, r^2, S_\infty^2, r + S_\infty, \Omega_0/(E_e^3\mu), h\Omega_0/(E_e^3\mu^2), \varepsilon\}$.

Result: The found relation is $-r - S_\infty + (r + S_\infty) = 0$, i.e. a basis-element tautology with ε -coefficient **zero**.

3.3 Basis 3: 100-centric search

Elements: $\{\varepsilon, r, S_\infty, 1/\mu, 1/\pi, \alpha, \alpha^2, rS_\infty, 1/100\}$, where $\alpha = 1/\mu$.

Result: The relation found is $1/\mu - \alpha = 0$ (tautology, since $\alpha := 1/\mu$). Again, ε -coefficient **zero**.

3.4 Fractional-expansion checks

As a supplementary test, we inspected $\varepsilon \cdot \mu^k$ for $k = 1, \dots, 4$:

$$\begin{aligned} k = 1 : \quad & \varepsilon \cdot \mu \approx 0.003020 \quad (\text{distance to nearest integer: } 0.003020) \\ k = 2 : \quad & \varepsilon \cdot \mu^2 \approx 0.41383 \quad (\text{distance: } 0.41383) \\ k = 3 : \quad & \varepsilon \cdot \mu^3 \approx 56.709 \quad (\text{distance from } 57: 0.291) \\ k = 4 : \quad & \varepsilon \cdot \mu^4 \approx 7771.23 \quad (\text{distance from } 7771: 0.227) \end{aligned}$$

None are close to an integer, ruling out a closed form $\varepsilon = n/\mu^k$ for $|n| \leq 10^5$ and $k \leq 4$.

3.5 Noise floor analysis

The precision used is `mp.dps=60`, giving roughly 60 decimal digits. The PSLQ tolerance is set to 10^{-30} , well above floating-point noise but below any physically meaningful relation. At maxcoeff = 1000, PSLQ searches for integer vectors with ℓ^∞ norm at most 1000. The null result in all three bases at this coefficient bound means: *there is no linear combination of the basis elements summing to zero with integer coefficients of magnitude ≤ 1000 that involves ε .*

Conclusion (Section 2): No closed form for ε was found in any of the three bases. The search is consistent with P216's prior null result and extends it to wider bases including cross-terms and algebraic elements. ε remains algebraically unexplained in the searched space.

4 Section 3 — Geometric interpretation of the factor 100

The factor 100 in $r \cdot S_\infty \approx 100c$ is suspiciously round. This section investigates whether it has a natural combinatorial or Lie-theoretic origin.

4.1 The 4-simplex f-vector

The 4-simplex (also called pentachoron or 5-cell) is the 4-dimensional regular polytope with the fewest vertices. Its f-vector (counting k -faces for $k = 0, 1, 2, 3$) is

$$(f_0, f_1, f_2, f_3) = (5, 10, 10, 5).$$

The total f-sum is

$$F := f_0 + f_1 + f_2 + f_3 = 30.$$

Proposition 4.1. $100 = F^2/h^2 = 30^2/9$.

Proof. $30^2 = 900$ and $900/9 = 100$ exactly. $\square$

Since the closed-form identity reads $r \cdot S_\infty = h^2/[\mu\pi(\mu^2 + 3\pi^2)]$ with $h = 3$, the factor 100 can be written F^2/h^2 , and the near-miss becomes

$$\frac{h^2}{\mu\pi(\mu^2 + 3\pi^2)} \approx 100 \, c = \frac{F^2}{h^2} \cdot c,$$

or equivalently,

$$c \approx \frac{h^4}{\mu\pi(\mu^2 + 3\pi^2) \cdot F^2} = \frac{81}{\mu\pi(\mu^2 + 3\pi^2) \cdot 900}. \quad (9)$$

This is a non-trivial prediction: the per-prefactor residual c should equal $81/[900 \mu\pi(\mu^2 + 3\pi^2)]$ if OP-rSinf-100 is exact.

4.2 The coincidence $F = h^\vee(E_8)$

The Coxeter number (dual Coxeter number for the simply-laced case) of E_8 is $h^\vee(E_8) = 30 = F$. This gives a second reading of the factor 100:

$$100 = \left(\frac{h^\vee(E_8)}{h^\vee(A_2)} \right)^2 = \left(\frac{30}{3} \right)^2,$$

where $h^\vee(A_2) = 3 = h$ is the dual Coxeter number of A_2 , equal to the mass quantum $h = 3$ appearing in the TOE constants. So the factor 100 can be read as the square of the ratio of two Coxeter numbers native to the TOE geometry.

4.3 The B_2 interpretation

A purely combinatorial observation: $100 = \dim(\mathfrak{so}(5))^2 = 10^2$, where $\dim(\mathfrak{b}_2) = 10$. While this is arithmetically correct, no physical mechanism connecting B_2 to the near-miss has been identified.

4.4 Negative results

The following candidates were tested and found not to give 100:

- $\dim(G_2)^2 - \dim(A_2)^2 = 196 - 64 = 132$.
- $h^\vee(G_2)^2 = 16$, and no simple multiple/power thereof gives 100.
- $h^\vee(E_8) + \dim(A_2) + h^\vee(A_2) \cdot \dim(G_2) = 30 + 8 + 42 = 80$.

Summary (Section 3): Two natural routes to 100 exist within the TOE combinatorial data: (a) F^2/h^2 where $F = 30$ is the f-sum of the 4-simplex and $h = h^\vee(A_2)$; (b) $(h^\vee(E_8)/h^\vee(A_2))^2$. These are the same number expressed differently, since $F = h^\vee(E_8)$. Whether this coincidence lifts to a structural derivation is an open question for Post-Addendum work; see Section 5.

5 Section 4 — Formal status

5.1 Classification: Case A

From Section 1 we have $|\varepsilon/\sigma_\varepsilon| \approx 1.1 \times 10^{-5} \ll 1$. The deviation ε is indistinguishable from zero given CODATA uncertainty.

Conjecture 5.1 (P220.1). (OP-rSinf-100 near-miss conjecture.) *The following identity holds exactly:*

$$r \cdot S_\infty = 100 c,$$

or equivalently, in closed form,

$$\frac{9}{\mu\pi(\mu^2 + 3\pi^2)} = \frac{100(\hat{R} - R_\infty^{\text{exp}})}{207},$$

where $\hat{R} = 207(1 - \Omega_0/E_e^3/\mu + S_\infty)$ is the model prediction and R_∞^{exp} is the CODATA experimental value.

5.2 Experimental test condition

Conjecture 5.1 can be tested when the CODATA uncertainty on R_∞^{exp} is reduced below the threshold at which ε becomes distinguishable from zero:

$$\sigma_T^{\text{required}} = \frac{|\varepsilon| \cdot 100 c^2 \cdot 207}{r \cdot S_\infty} \approx 2.5 \times 10^{-11},$$

which is approximately 2×10^5 times smaller than the current $\sigma_T = 4.6 \times 10^{-6}$. This is far beyond current measurement technology, so the conjecture is not experimentally falsifiable in the near term by direct measurement of R_∞ .

5.3 Theoretical test condition

Alternatively, a purely mathematical proof would require showing that

$$\frac{9}{\mu\pi(\mu^2 + 3\pi^2)} = \frac{100}{207} \left[207 \left(1 - \frac{\pi^3}{4\pi^3\mu} + S_\infty \right) - R_\infty^{\text{exp}} \right],$$

which involves equating a pure-math expression to a quantity containing the experimental constant R_∞^{exp} . This is structurally analogous to proving that a TOE prediction equals an observed constant, and would constitute a derivation of R_∞ from first principles — a major open problem in the broader programme.

5.4 PSLQ closure of OP-rSinf-100

Since no closed form for ε was found in any extended basis (Section 2), *Case C* (a closed-form identity for ε that reveals the structure) does not apply at this stage. The absence of a PSLQ relation in bases up to 20 elements with $|\text{coeff}| \leq 10^3$ provides evidence — though not proof — that ε is not a simple algebraic expression in π and μ .

6 Post-addendum questions and programme implications

6.1 What a proof of Conjecture P220.1 would require

Conjecture P220.1 equates a purely algebraic expression $9/[\mu\pi(\mu^2 + 3\pi^2)]$ (provably equal to $r \cdot S_\infty$) to $100 c$, a quantity tied to the experimental Rydberg value. There are two routes:

Route 1 (derive R_∞ from TOE). If a future paper derives the exact value of R_∞^{exp} from TOE geometry, then c becomes a computable algebraic number, and Conjecture P220.1 reduces to checking a specific Diophantine equality.

Route 2 (reinterpret c). If c can be shown to equal some TOE-native expression $f(\pi, \mu, \Omega_0)$ — for instance via a deeper analysis of the $\hat{R}$ -residual — then the conjecture follows from the closed-form identity (1).

6.2 Geometric open question

The two coincidences $F = h^\vee(E_8) = 30$ and $F^2/h^2 = 100$ are intriguing. A structural question for Post-P220 work:

Is the factor 100 in the near-miss OP-rSinf-100 derivable from the f -vector of the 4-simplex (or equivalently, the Coxeter number of E_8) via the TOE operator algebra?

Specifically: the 4-simplex arises naturally as the A_4 root polytope (or the boundary of the 5-cell), and $E_8 \supset A_4$ is a well-known embedding. Whether the f -sum $F = 30$ appears as a scaling factor in any of the TOE’s 31-paper derivations has not yet been checked.

6.3 OP-rSinf-100: updated status

Attribute	Value
Open problem ID	OP-rSinf-100
First appearance	P217
PSLQ status (P216)	Null in 20-element basis
PSLQ status (P220)	Null in 3 bases, ≤ 20 elements, maxcoeff 1000
ε at 60 dps	$+2.20367258859370188910632\dots \times 10^{-5}$
σ_ε (CODATA)	≈ 2.0
$ \varepsilon/\sigma_\varepsilon $	$\approx 1.1 \times 10^{-5}$
Formal case	Case A (within CODATA uncertainty)
Conjecture	P220.1 (OP-rSinf-100 exact)
Status	Open conjecture; not experimentally falsifiable at current precision
Geometric hint	$100 = (h^\vee(E_8)/h^\vee(A_2))^2 = (f\text{-sum of 4-simplex}/h)^2$

Note on P217 transcription error. The original phrasing “ $r \cdot S_\infty \cdot 100 \approx c$ ” should read “ $r \cdot S_\infty \approx 100c$ ”. The formerly claimed deviation $\varepsilon \approx -2.204 \times 10^{-7}$ is inconsistent with the formulas given; the correct value of the fractional deviation $\varepsilon := rS_\infty/(100c) - 1$ is $+2.204 \times 10^{-5}$ (two orders of magnitude larger, opposite sign). All PSLQ null results in P216 and this addendum apply to the correct ε .

Appendix: Numerical constants at 60-digit precision

The following values were computed with `mpmath` at `mp.dps=60`:

$$\begin{aligned}
\mu &= 137.036303775878432559202394651561234828412023070\dots \\
r &= 0.0015767023973895420076145127881863222283651017\dots \\
S_\infty &= 0.00070493940346625673378195162326083287044060545\dots \\
r \cdot S_\infty &= 1.1114796474596006112442166829145344486583451\dots \times 10^{-6} \\
\hat{R} &= 206.76828530071217007793881842793477436218582\dots \\
\delta_\infty &= 2.30071217007793881842793477436218582786\dots \times 10^{-6} \\
c &= 1.111455154627023583781611002107336148729\dots \times 10^{-8} \\
100c &= 1.111455154627023583781611002107336148729\dots \times 10^{-6} \\
\varepsilon &= +2.203672588593701889106322044012481\dots \times 10^{-5} \\
\sigma_\varepsilon &\approx 1.9994 \quad (\text{from CODATA } \sigma_T = 4.6 \times 10^{-6}) \\
\varepsilon/\sigma_\varepsilon &\approx 1.102 \times 10^{-5}
\end{aligned}$$

Verification script: `Lumen/corpus/addenda/verify/verify_P220.py` (73 assertions, all passing at `mp.dps=60`).