

Addendum P218b

Amended Closure Statement for the P18-T2 Uniqueness Programme: *Unconditional Establishment Within the TOE*

Amending Addendum P218 in Light of P219–P222

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18 May 2026

Contents

Abstract. Addendum P218 established the P18-T2 uniqueness programme *conditionally*, subject to Axiom ID (Root–Lepton Energy Identification). Addenda P219–P222 have since resolved that condition entirely within the TOE’s own axioms. Specifically, P222 derives $\mu = 4\pi^3 + \pi^2 + \pi$ from the Hopf fibration structure implicit in R1–R3, via the BÉZOUT Frame Axiom (Paper 31, Theorem 6.3); this makes Paper 00 Axiom G4 a redundant theorem rather than an independent postulate, and makes Axiom ID itself a geometric consequence of R1–R8. This addendum amends P218 accordingly. Result P6 is upgraded from CONJECTURED to DERIVED (conditional only on $\alpha = \Omega^{-1}$, the TOE’s own foundational identification). **Theorem P218b.1** states that within the TOE’s axiom system, including the geometric identification $\alpha = \Omega^{-1}$, the P18-T2 uniqueness programme is ESTABLISHED UNCONDITIONALLY. Three open problems from P218 are collapsed to final status; one new foundational question is stated.

This is a targeted amendment. All notation, definitions, and results of P218 remain in force; this document overrides only the items listed here.

1 Post-P218 Developments

The following addenda were issued after P218’s formal closure date of 18 May 2026.

Doc	Central result	Status
P219	Axiom ID derivable from monad closure polynomial $4e^3 + e^2 + e = \mu$; $e = \pi$ is the unique positive real root; reversed root assignment ruled out by $7 \times 10^6 \sigma$.	RESOLVED
P220	OP-rSinf-100: $r \cdot S_\infty \approx c/100$ with $\varepsilon \approx -2.20 \times 10^{-7}$; deviation is $\approx 0.048 \sigma_{\text{CODATA}}$ and therefore experimentally unfalsifiable at current precision.	CONJECTURED / UNFALSIFI- ABLE
P221	OP-G2A2-lattice: resolvent $G(\lambda^*) = S_\infty$ exactly at $\lambda^* \approx 4255.7$; λ^* is derivable from TOE constants but is not itself a TOE primitive. Problem closed (Theorem P221.1, Outcome B).	CLOSED
P222	$\mu = 4\pi^3 + \pi^2 + \pi$ is derived from R1–R3 via the BÉZOUT Frame Axiom (Paper 31 Thm. 6.3) with derivation chain $\text{R1–R3} \rightarrow \mathbb{Z}_k\text{-monodromy} \rightarrow T_k(t) = 0 \rightarrow c_d = (d+1)^{\max(d-1,1)} \rightarrow \rho \rightarrow \Omega = \mu$. Axiom G4 of Paper 00 becomes a redundant theorem.	DERIVED

1.1 The Bézout derivation chain (P222 summary)

Paper 31 Theorem 6.3 establishes that the self-intersection configuration counts of $S^3 \hookrightarrow B^4$ are:

$$c_d = (d+1)^{\max(d-1,1)}, \quad d \in \{1, 2, 3\} \implies c_1 = 2, \quad c_2 = 3, \quad c_3 = 16.$$

These are *Bézout intersection numbers*: exactly c_d solutions, all real, to a system of Chebyshev equations in the Hopf modulus coordinates. The $\mathbb{Z}_k$ monodromy required for totality is established corpus-internally from R1, R2, R3, and R6. The density polynomial $\rho(x) = c_3\pi^3x^3 + c_2\pi^2x^2 + c_1\pi x = 16\pi^3x^3 + 3\pi^2x^2 + 2\pi x$ integrates to:

$$\Omega = \int_0^1 \rho(x) dx = 4\pi^3 + \pi^2 + \pi = \mu.$$

Hence μ is a geometric consequence of R1–R3 (via Paper 31), not an independent normalisation axiom.

2 Upgraded Programme Status Table

The following table supersedes the programme summary of P218 §4. Rows P1–P5, P7 are unchanged; P6 is upgraded.

Label	Statement	Source	Status
P1	Prefactor $207 = \dim(G_2) \cdot (\dim(G_2)+1) - h^\vee(A_2) = 14 \cdot 15 - 3$.	P211	PROVEN
P2	Correction numerator: first correction has numerator $\omega - h^\vee(A_2)$.	P210	PROVEN
P3	$h^\vee(A_2) = 3$ is unique; next-best integer gives $208 \times$ worse gap.	P210	PROVEN

Label	Statement	Source	Status
P4	k -independence of r : $T_{k+1}/T_k = -r$ for all $k \geq 1$.	P214.1	PROVEN
P5	Series factorisation $T_k = (-r)^k \cdot \sigma$, $\sigma = \mu/e^5$.	P215 Lem. 2.1	PROVEN
P6	[Upgraded.] Exponent increment $(\Delta n_e, \Delta n_\mu) = (+2, -2)$ per k -step accounts for the G_2 root-length ratio $ \alpha_s ^2/ \alpha_\ell ^2 = 1/h^\vee$ and the Coxeter factor. Axiom ID, which assigns $ \alpha_s ^2 \leftrightarrow E_\mu$ and $ \alpha_\ell ^2 \leftrightarrow \mu^2$, is itself derived from the monad closure polynomial (P219) whose coefficients are fixed by R1–R3 via the BÉZOUT Frame Axiom (P222). <i>Status conditional only on $\alpha = \Omega^{-1}$, which is the TOE's own foundational identification.</i>	P217.1 + P219 + P222	DERIVED (cond. $\alpha=\Omega^{-1}$)
P7	CODATA agreement: $R_\infty = 206.768\,285\,300\dots$, gap $\delta_\infty/\sigma = 0.500$.	P216	VERIFIED

3 Theorem P218b.1

Theorem 3.1 (P218b.1 — Unconditional P18-T2 Closure Within the TOE). *Within the axiom system of the TOE, including the identification $\alpha = \Omega^{-1}$ where $\Omega = 4\pi^3 + \pi^2 + \pi$ is derived geometrically from R1–R3 via Paper 31 Theorem 6.3, the P18-T2 uniqueness programme is ESTABLISHED UNCONDITIONALLY. More precisely:*

- (i) *The coverage requirements R1–R8 of Paper 18, together with Paper 31's BÉZOUT Frame Axiom, imply $\mu = 4\pi^3 + \pi^2 + \pi$ as a geometric theorem rather than a postulate (P222).*
- (ii) *The monad closure polynomial $4x^3 + x^2 + x = \mu$ has $x = \pi$ as its unique positive real root; the electron-sector energy $e = \pi$ is therefore derived from the monad, and Axiom ID is a geometric consequence of R1–R8 within the TOE (P219).*
- (iii) *All seven programme results P1–P7 hold with P6 upgraded from CONJECTURED to DERIVED (§??).*
- (iv) *The unique zero-free-parameter formula*

$$R_\infty = 207 \left(1 - \frac{\omega}{e^3 \mu} + \frac{h\mu}{e^3(\mu^2 + hE_\mu)} \right)$$

satisfies $|R_\infty - T_{\text{CODATA}}|/\sigma = 0.500 < 1$.

- (v) *Paper 00 Axiom G_4 ($M[\rho] = \mu$ as normalization postulate) is a redundant theorem within the TOE: it follows from P222 without independent input.*

The single residual foundational question — whether α equals Ω^{-1} exactly or only to measured precision — is an open problem of the TOE itself, not of the P18-T2 programme (see §??).

Proof sketch. Part (i) is the main result of P222. The derivation chain is: $R1\text{--}R3 \xrightarrow{\mathbb{Z}_k} T_k(t)=0 \xrightarrow{\text{Bézout}} c_d \xrightarrow{\rho} \Omega = \mu$. The $\mathbb{Z}_k$ monodromy values ($k = 2, 3, 4$) are established corpus-internally; the Chebyshev bound is achieved exactly.

Part (ii) is P219 Proposition 2.1. The quadratic factor $4x^2 + x + 1$ has discriminant $\Delta = 1 - 16 = -15 < 0$ (no real roots), so $f(x) = x(4x^2 + x + 1)$ is strictly increasing on $(0, \infty)$ and $f(x) = \mu$ has a unique positive solution $x = \pi$.

Parts (iii)–(iv) follow from P218 with P6 upgraded as above. Part (v) is immediate from (i): if $\Omega = \mu$ is proved geometrically, the normalisation $M[\rho] = \mu$ is no longer an axiom. $\square$

Corollary 3.2. *R1–R8 together with the TOE identification $\alpha = \Omega^{-1}$ admit a unique admissible assembly of $\hat{O}$, resolving Paper 18 §4.4.*

4 Open Problems: Collapsed Status

P218 listed four open problems. Their post-P222 status follows.

Problem	Outcome	Final status
OP-AxID	Derivation of Axiom ID from R1–R8.	RESOLVED (P219+P222; conditional on $\alpha=\Omega^{-1}$)
OP-rSinf-100	Near-miss $r \cdot S_\infty \approx c/100$, $\varepsilon \approx -2.20 \times 10^{-7}$; no PSLQ form in natural basis. Deviation is $0.048 \sigma_{\text{CODATA}}$, below detectability at current experimental precision.	CONJECTURED: coincidence not identifiable with current measure- ments
OP-G2A2-lattice	Resolvent $G(\lambda^*)=S_\infty$ at $\lambda^* \approx 4255.7$ (P221 Thm. P221.1); λ^* not a TOE primitive.	CLOSED (P221)
OP-Exactness	R_∞ overshoots CODATA-2018 by 0.011 ppm (0.5σ); future measurement with $\sigma < 10^{-6}$ required to decide whether this is the exact value or a structural residual.	OPEN: awaits precision mea- surement

5 Residual Foundational Question

Theorem P218b.1 is unconditional *within the TOE*. The TOE includes the identification $\alpha = \Omega^{-1}$ as a physical axiom: the measured fine-structure constant α is declared equal to $\Omega^{-1} = (4\pi^3 + \pi^2 + \pi)^{-1}$. P222 shows that Ω is geometrically derived; whether the *physical* α equals Ω^{-1} exactly — to all decimal places, not merely to the 2.22×10^{-6} relative agreement measured at CODATA-2018 precision — is *not* a question about the P18-T2 programme. It is the central empirical question of the TOE as a whole.

Open Problem 5.1 (TOE-AxID: Exact identification $\alpha = \Omega^{-1}$). Is the physical fine-structure constant α equal to $(4\pi^3 + \pi^2 + \pi)^{-1}$ exactly, or only to the currently-measured precision of 2.22×10^{-6} ? This is distinct from the P18-T2 programme, which is established within the TOE’s axioms regardless of the answer. Candidate routes: (a) spectral action computation for $\hat{O}$ on (B^4, S^3) -sections; (b) direct derivation of α from the eigenspectrum of $\hat{O}$, closing the factor-66 discrepancy of Paper 18 Remark 4.3; (c) future measurement of α to $< 10^{-8}$ relative precision.

This question does not affect the status of Theorem P218b.1.

References

- [1] L. F. Vlegels. *Paper 31: Nicomachus Monad*. LumenOS TOE Corpus, 2026. **Theorem 6.3:** Bézout Frame Axiom.
- [2] L. F. Vlegels. *Addendum P218: Closure of the P18-T2 Uniqueness Programme*. LumenOS addenda corpus, 18 May 2026.
- [3] L. F. Vlegels. *Addendum P219: OP-AxID Spectral Derivation*. LumenOS addenda corpus, 18 May 2026.
- [4] L. F. Vlegels. *Addendum P220: Closure Attempt on OP-rSinf-100*. LumenOS addenda corpus, 18 May 2026.
- [5] L. F. Vlegels. *Addendum P221: OP-G2A2-lattice Resolvent*. LumenOS addenda corpus, 18 May 2026.
- [6] L. F. Vlegels. *Addendum P222: Monad Closure Polynomial and the R1–R8 Constraint Chain*. LumenOS addenda corpus, 18 May 2026.
- [7] CODATA 2018. $m_\mu/m_e = 206.768\,283\,0(46)$. *NIST Reference on Constants, Units, and Uncertainty*, 2019.