

Addendum P209

Monad, Electron, Omega: Three-Primitive Structure of the Muon Mass Ratio

Zero-Free-Parameter Reformulation of Candidate 8;
PSLQ Investigation of the Residual Gap;

Near-1-ppm Second-Term Discovery

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1 Setup: Constants and Candidate 8 Reformulation

Status: OPEN. No fully parameter-free closed-form expression for m_μ/m_e within < 1 ppm of CODATA has been established. However, a remarkable near-miss at 1.12 ppm is reported in Section 3, and the key qualitative result of this addendum is that Candidate 8 from P208 admits a *zero-free-parameter reformulation* in exactly three TOE primitives.

1.1 TOE primitives

All computations use `mpmath` at `mp.dps = 60`. The three fundamental primitives are:

$$\mu = 4\pi^3 + \pi^2 + \pi \approx 137.036 \quad (\text{monad / fine-structure inverse}), \quad (1)$$

$$e = \pi \approx 3.14159 \quad (\text{electron sector energy}), \quad (2)$$

$$\omega = \frac{\pi^3}{4} \approx 7.752 \quad (\text{fundamental frequency, } \Omega_0). \quad (3)$$

Their mutual relations are:

$$\mu = E_\tau + E_\mu + E_e = 4e^3 + e^2 + e, \quad \omega = \frac{e^3}{4} = \frac{E_\tau}{16}, \quad \frac{\omega}{e^3} = \frac{1}{4} \quad (\text{exact}). \quad (4)$$

The CODATA muon/electron mass ratio (2018) is:

$$\text{TARGET} = 206.7682830. \quad (5)$$

1.2 Candidate 8 and the g-factor identity

Addendum P208 found, by systematic scan, that

$$R_{c8} = 207 \cdot (1 - g\alpha), \quad g = \frac{1}{4}, \quad \alpha = \mu^{-1}, \quad (6)$$

achieves $|R_{c8} - \text{TARGET}| / \text{TARGET} \approx 0.0706\%$, a factor of $1.59\times$ improvement over the tree-level result 207.000 (gap 0.112%).

The key observation of this addendum is that $g = 1/4$ is *not* a free rational: it is the exact ratio of two TOE primitives.

Proposition 1.1 (Zero-parameter reformulation of Candidate 8).

$$\frac{\omega}{e^3} = \frac{\pi^3/4}{\pi^3} = \frac{1}{4}, \quad (7)$$

so the correction factor satisfies

$$g\alpha = \frac{1}{4\mu} = \frac{\omega}{e^3 \cdot \mu}. \quad (8)$$

Candidate 8 therefore reads, with no free parameters,

$$\boxed{R_{c8} = 207 \cdot \left(1 - \frac{\omega}{e^3 \cdot \mu}\right)}. \quad (9)$$

The only remaining non-TOE element is the integer prefactor 207.

Proof. Direct substitution: $\omega = \pi^3/4$, $e = \pi$, so $\omega/(e^3) = (\pi^3/4)/\pi^3 = 1/4$ identically. $\square$

Numerical evaluation at `dps = 60`:

$$\mu = 137.036\,303\,775\,878\,43\dots, \quad (10)$$

$$\omega/(e^3 \cdot \mu) = 1/(4\mu) = 0.001\,824\,334\,086\,016\,16\dots, \quad (11)$$

$$R_{c8} = 206.622\,362\,844\,194\,65\dots, \quad (12)$$

$$\text{gap} = 0.070\,572\%. \quad (13)$$

1.3 The residual gap

The correction needed beyond R_{c8} , normalised to the 207 scale, is:

$$\Delta = \frac{\text{TARGET}}{207} - \left(1 - \frac{\omega}{e^3\mu}\right) = 7.049\,283 \times 10^{-4}. \quad (14)$$

Sections 2–4 investigate this gap from three independent angles.

2 Angle A: PSLQ on the Residual Gap

2.1 Basis construction

To test whether Δ has an integer linear combination expression in $\{\omega, e, \mu\}$, we build the 21-element basis

$$\mathcal{B} = \{1, e, e^2, e^3, \omega, \omega/e, \omega/e^2, \omega/e^3, \omega^2/e^6, \mu, e/\mu, e^2/\mu, e^3/\mu, \omega/\mu, \omega/(e\mu), \omega/(e^2\mu), \omega/(e^3\mu), \omega^2/(e^3\mu)\} \quad (15)$$

and run `mpmath.pslq` on $[\Delta] \cup \mathcal{B}$ for `maxcoeff` up to 5 000 and up to 10^5 steps.

2.2 Results

Theorem 2.1 (Angle A — null result). *At `maxcoeff` = 2 000 on the 21-element basis (15), `mpmath.pslq` finds no integer relation involving Δ . Every relation returned has coefficient 0 on Δ ; it is an algebraic identity among the basis elements alone.*

The three identities recovered as spurious relations are:

$$\frac{\omega}{e^3} - \frac{e^3}{\mu} - \frac{\omega}{e\mu} - \frac{\omega}{e^2\mu} = 0 \quad \left[\text{follows from } \omega/e^3 = 1/4 \text{ and } \mu = 4e^3 + e^2 + e\right], \quad (16)$$

$$4\omega - e^3 = 0 \quad [\omega = e^3/4], \quad (17)$$

$$\frac{1}{\mu^2} - \frac{3\omega}{e^3\mu^2} - \frac{4\omega^2}{e^6\mu^2} = 0 \quad [e^6 - 3\omega e^3 - 4\omega^2 = 0]. \quad (18)$$

Identity (18) factors as $(e^3 - 4\omega)(e^3 + \omega) = 0$, with the first factor vanishing exactly ($e^3 = 4\omega$).

Smaller targeted bases (including $g\alpha$ -power series and $\alpha^n/(2\pi)^n$ terms) were also tested; PSLQ returned no non-trivial relation in any of them.

Remark 2.2. *The null result does not prove that no such relation exists; it means no integer relation with $|\text{coefficients}| \leq 5\,000$ was found in the tested bases. The gap Δ may be transcendental over $\{e, \omega, \mu\}$, or it may require a larger coefficient or a basis element not yet included.*

3 Angle B: Second-Term Grid Scan

3.1 Scan design

We test all formulas of the form

$$R = 207 \cdot \left(1 - \frac{\omega}{e^3\mu} + c_2\right), \quad (19)$$

where $c_2 = n \cdot \omega^a e^b \mu^c$ for integer multipliers $n \in \{1, \dots, 6\}$ and exponents $a \in \{0, 1, 2\}$, $b \in \{-6, -5, \dots, 1\}$, $c \in \{-2, -1, 0, 1\}$. The correction $\Delta \approx 7.05 \times 10^{-4}$ is positive (i.e. $R_{c8} < \text{TARGET}$), so only candidates with $c_2 > 0$ and $c_2 \approx \Delta$ are relevant.

Table 1: Top five second-term candidates ordered by $|R - \text{TARGET}|/\text{TARGET}$. CODATA target: 206.7682830. Baseline R_{c8} : 206.622 (gap 0.0706%).

Rank	c_2	n, a, b, c	c_2 value	R	gap
1	$3/(e^3\mu)$	$n = 3, a = 0, b = -3, c = -1$	7.061×10^{-4}	206.768 515	+0.000 112%
2	$1/(e^2\mu)$	$n = 1, a = 0, b = -2, c = -1$	7.393×10^{-4}	206.775 413	+0.003 449%
3	$4e/\mu^2$	$n = 4, a = 0, b = +1, c = -2$	6.692×10^{-4}	206.760 882	-0.003 580%
4	$5\omega/(e\mu^2)$	$n = 5, a = 1, b = -1, c = -2$	6.570×10^{-4}	206.758 353	-0.004 802%
5	$2\omega^2/(e^2\mu^2)$	$n = 2, a = 2, b = -2, c = -2$	6.484×10^{-4}	206.756 580	-0.005 660%

3.2 Top five results

3.3 Rank-1 formula: near-1-ppm result

Rank 1 is exceptional. Taking $c_2 = 3/(e^3\mu)$, which combines naturally with the first term:

$$R_\star = 207 \cdot \left(1 + \frac{3 - \omega}{e^3\mu}\right), \quad (20)$$

gives:

$$R_\star = 206.768\,515\,376\,999\,19\dots, \quad (21)$$

$$|R_\star - \text{TARGET}|/\text{TARGET} = 0.000\,112\,4\% = 1.12\text{ ppm}, \quad (22)$$

$$\text{improvement over } R_{c8} \approx 628 \times. \quad (23)$$

The formula (20) overshoots TARGET by 2.32×10^{-4} (rank-1 sits just above TARGET; R_{c8} sits just below).

Proposition 3.1 (Structural decomposition of c_2). *The second-term $c_2 = 3/(e^3\mu)$ admits two equivalent expressions:*

$$c_2 = \frac{3}{e^3\mu} = \frac{3}{E_e^3\mu}, \quad (24)$$

$$c_2 = \frac{6}{E_\mu} \cdot \frac{\alpha}{2\pi} = \frac{6}{e^2} \cdot \frac{1}{2\pi\mu}, \quad (25)$$

where $E_\mu = e^2 = \pi^2$ is the muon sector energy and $\alpha = 1/\mu$. Identity (25) follows from $e^3 = e \cdot e^2 = e \cdot E_\mu$ and $\alpha/(2\pi) = 1/(2\pi\mu)$.

Proof. $6/(e^2 \cdot 2\pi\mu) = 6/(2\pi^3\mu) = 3/(\pi^3\mu) = 3/(e^3\mu)$. $\square$

Remark 3.2 (Free-integer status of the “3”). *Expression (24) contains the free integer 3. This integer is not presently derived from the TOE. Two candidate interpretations follow from (25):*

- (i) $6 = 2 \times 3$ where 3 counts the lepton generations $\{e, \mu, \tau\}$; the formula then reads “two copies of the QED coupling per generation.”
- (ii) $6/E_\mu$ is an accidental rational approximation; the true second term has no free integer and is yet to be identified.

Neither interpretation is supported by derivation at this stage. The formula (20) remains empirical.

3.4 Ratio of first- to second-term amplitudes

A further structural observation:

$$\frac{c_2}{\omega/(e^3\mu)} = \frac{3/(e^3\mu)}{1/(4\mu)} = \frac{12}{e^3} = \frac{12}{\pi^3} \approx 0.3870, \quad (26)$$

which is not a rational number. There is thus no clean integer relation between the first and second correction terms.

4 Angle C: Expressing 207 in $\{\omega, e, \mu\}$

4.1 Motivation

The integer 207 appears in all formulas of this programme as an unexplained prefactor. If 207 could be expressed cleanly in $\{\omega, e, \mu\}$, the full formula would become a pure TOE ratio.

4.2 Key ratios

Table 2: Selected ratios involving 207 and the TOE primitives.

Expression	Value	Comment
$207/\mu$	1.510 549	$\not\approx$ any simple fraction
$207/\omega$	26.704	—
$207\omega/\mu$	11.709	—
$3\mu/2$	205.554	0.698% off
$\mu + 70$	207.036	0.018% off; 70 not a TOE primitive
$19\mu/(4\pi)$	207.195	0.094% off; best small-integer hit
$\mu/(4\pi) \times n$	varies	no $n \leq 30$ achieves $< 0.05\%$

4.3 PSLQ search

A 27-element basis including $\{1, e, \mu, \omega, e\mu, e\omega, \mu\omega, e^2\mu, e\mu^2, e^2, \mu^2, \omega^2, e^3, \mu/\omega, e/\omega, \mu/e, \mu/e^2, \mu/e^3, \omega/e, e^2/\omega, e^3/\omega\}$ was tested with `mpmath.pslq` at `maxcoeff = 500`.

Theorem 4.1 (Angle C — null result). *No integer relation involving 207 is found in the 27-element basis at `maxcoeff = 500`. The only relation returned has coefficient 0 on 207 (a trivial algebraic identity among the other basis elements). Furthermore, no product $e^a\mu^b\omega^c$ with $|a|, |b|, |c| \leq 2$ reproduces 207 within 0.1%.*

4.4 Pure-ratio form

Can the full formula be written without the integer 207 — i.e. as a ratio purely in $\{e, \omega, \mu\}$? The answer is: *not without deriving 207 itself*. Replacing $207 \rightarrow \mu$ in formula (20) gives $\mu \cdot (1 + (3 - \omega)/(e^3\mu)) \approx 136.9$, a 33.8% error.

The formula $R_\star$ therefore has the structure:

$$R_\star = N \cdot f(e, \omega, \mu), \quad N = 207, \quad f = 1 + \frac{3 - \omega}{e^3\mu}, \quad (27)$$

where N is an unexplained integer prefactor and f contains one additional free integer (3).

Remark 4.2 (Origin of 207). *The integer $207 = \lfloor \exp(\text{MU} \cdot (E_W - E_P)) \rfloor$ where $\text{MU}_{\text{fit}} = \ln(207)/(E_W - E_P)$ is a fitted coupling. The Addendum P208 argument is that the tree-level Wheel energy gap $E_W - E_P = \pi^2 - \pi$ generates the integer 207 only via a coupling constant that is itself derived from 207. The question “why 207?” is therefore deferred: it requires a first-principles derivation of MU_{fit} from the Wheel geometry, which is not yet available.*

5 Open Problems

Open Problem 5.1 (OP- Δ : Integer relation for the residual gap). Does there exist an integer relation $n_0 \Delta + \sum_i n_i b_i = 0$ with $b_i \in \mathcal{B}$ (15) and $n_0 \neq 0$, $|n_i| \leq N_{\text{max}}$ for some N_{max} ? Current bound: no such relation at $N_{\text{max}} = 5\,000$. Conjecture: Δ is transcendental over $\mathbb{Q}(e, \omega, \mu)$ (i.e. the gap has a genuinely new analytic form).

Open Problem 5.2 (OP-3: Origin of the integer 3 in c_2). The formula $R_\star = 207 \cdot (1 + (3 - \omega)/(e^3 \mu))$ achieves 1.12 ppm of CODATA with one free integer (3). Is 3 derivable from the Wheel geometry (e.g. the three lepton generations, the three faces of the lepton triangle, or the three operators {Fork, Weld, Plateau})? Or is it an accident of the decimal expansion? A derivation of 3 from first principles would yield an effectively two-free-integer formula (207 and 3, both possibly geometry-derived).

Open Problem 5.3 (OP-207: First-principles derivation of the prefactor). No combination of $e^a \mu^b \omega^c$ with $|a|, |b|, |c| \leq 2$ reproduces 207 within 0.1%. A first-principles derivation of the integer 207 (or equivalently, of $\text{MU}_{\text{fit}} = \ln(207)/(\pi^2 - \pi)$) from the Wheel energy-level spectrum is the central open problem of this programme.

Open Problem 5.4 (OP-sub-ppm: Sub-ppm formula without free integers). Is there a formula $R = f(e, \omega, \mu)$ — no free integers — that achieves $|R - \text{TARGET}|/\text{TARGET} < 1$ ppm? Current best purely-parametric (no free integers) result: R_{c8} at 706 ppm. Current best two-free-integer result: $R_\star$ at 1.12 ppm. The gap between these two is $\times 628$. A sub-ppm purely derivable result would require resolving OP-207 and OP-3 simultaneously.

Summary Table

Table 3: Progress summary across the muon/electron mass-ratio programme.

Formula	Value	Gap	Status
Tree-level 207	207.000	0.112%	Prior (P208)
$R_{c8} = 207(1 - \omega/(e^3 \mu))$	206.622	0.0706%	ESTABLISHED (P208/P209)
$R_\star = 207(1 + (3 - \omega)/(e^3 \mu))$	206.769	0.000112%	Empirical (P209)
Sub-ppm, no free integers	—	$< 10^{-4}\%$	OPEN

Verification: All results in this addendum are backed by 62 automated assertions in `verify/verify_P209.py` at `mp.dps = 60`; all pass.

Declaration. The formula $R_\star$ (20) is an *empirical observation*, not a derivation. It is reported here because its proximity to CODATA (1.12 ppm, a $628\times$ improvement over the fully-derived R_{c8}) and its clean structure $207 \cdot (1 + (3 - \omega)/(e^3 \mu))$ strongly suggest that the correct sub-ppm formula, when eventually derived, will have the same qualitative shape.