

Addendum 197: PSLQ Grounding and Mass Formula Convergence

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Abstract

Addenda P193–P196 identified five geometric routes producing near-misses to the muon-to-electron mass ratio $m_\mu/m_e \approx 206.7682830$ (PDG). This note performs high-precision numerical grounding for those five routes, extracts the exact coefficient c_{ex} in the formula $\alpha^{-1} + c\Omega_0$, tests a one-loop G_2 correction, and states a convergence conjecture. The problem remains **open** but carries genuine geometric structure: three independent S^3 spectral constructions converge on a cluster of width ~ 0.05 centred on the PDG value.

1 Introduction

Fix the TOE constants

$$\alpha^{-1} = 4\pi^3 + \pi^2 + \pi, \quad \Omega_0 = \frac{\pi^3}{4}, \quad T = \pi \alpha^{-1}, \quad (1)$$

with layer fractions $f_e = \frac{1}{2}$, $f_\partial = \frac{3}{10}$, $f_b = \frac{1}{5}$, and $\alpha = 1/\alpha^{-1}$. The target mass ratio is

$$R = \frac{m_\mu}{m_e} = 206.7682830 \quad (\text{PDG 2024}).$$

Table 1 ranks the five near-miss routes found in P193–P196 by closeness to R .

Table 1: Five near-miss routes to m_μ/m_e .

Rank	Route	Value	Gap $ v - R $	Rel. %
1	Dirac ² : $(417/29)^2$	206.764566	0.003717	0.0018%
2	G_2 +loop: $207(1 - \alpha/2\pi)$	206.759589	0.008694	0.0042%
3	CR/Hopf: $827/4$	206.750000	0.018283	0.0088%
4	$\alpha^{-1} + 9\Omega_0$	206.800426	0.032143	0.0156%
5	G_2 NNLO: $14 \times 15 - 3$	207.000000	0.231717	0.1121%
6	$\lambda_5^{3/2}$: $35^{3/2}$	207.062792	0.294509	0.1424%

2 Coefficient analysis for $\alpha^{-1} + c\Omega_0$

2.1 Exact coefficient

The formula $R \approx \alpha^{-1} + c\Omega_0$ defines the exact coefficient

$$c_{\text{ex}} = \frac{R - \alpha^{-1}}{\Omega_0} = \frac{4(R - 4\pi^3 - \pi^2 - \pi)}{\pi^3}. \quad (2)$$

Evaluated at 40-digit precision,

$$c_{\text{ex}} = 8.99585331617\dots \quad (3)$$

This is tantalizingly close to 9 but does not equal 9; the deficit is

$$\delta = 9 - c_{\text{ex}} = 0.004146684\dots \quad (4)$$

2.2 Candidate decomposition of δ

We tested all small rational combinations of $\{\alpha, \pi, \Omega_0/\alpha^{-1}\}$ as candidates for δ . The closest TOE expression found is $f_b \cdot \alpha = \frac{1}{5} \cdot \alpha \approx 0.001459$, which falls 65% short. No closed-form TOE expression for δ is available at present.

Remark 2.1. *The natural candidate $9\alpha \approx 0.06568$ is $\sim 15\times$ too large; $\alpha\pi \approx 0.02293$ is $\sim 5.5\times$ too large. The residual $\delta \approx 4.1 \times 10^{-3}$ has no clean factorisation in terms of the standard TOE operators. It may signal a next-order Hopf-degree coupling not yet derived from first principles (see OP-ID, §7).*

3 The G_2 NNLO formula and the one-loop bridge

3.1 Tree level

The G_2 structure constant count gives the tree-level prediction

$$\dim(G_2)(\dim(G_2) + 1) - \dim(A_2) = 14 \times 15 - 3 = 207.$$

The gap $207 - R \approx 0.232$ is large enough that the tree formula alone is not a useful approximation.

3.2 One-loop QED correction

A natural one-loop correction to any mass ratio in a $U(1)$ -gauged theory is of order $\alpha/(2\pi)$:

$$\frac{\alpha}{2\pi} = \frac{1}{2\pi\alpha^{-1}} = \frac{1}{2\pi(4\pi^3 + \pi^2 + \pi)} \approx 1.1614 \times 10^{-3}. \quad (5)$$

The required fractional correction to close the G_2 gap is

$$\varepsilon = 1 - \frac{R}{207} \approx 1.1194 \times 10^{-3}.$$

The two agree to within 3.8%.

Theorem 3.1 (G_2 + one-loop).

$$207\left(1 - \frac{\alpha}{2\pi}\right) = 206.7596\dots$$

The gap from the PDG value is $|206.7596 - R| = 0.00869$, a relative deviation of 0.0042%.

Proof. Direct substitution of $\alpha^{-1} = 4\pi^3 + \pi^2 + \pi$ into $207(1 - 1/(2\pi\alpha^{-1}))$ and evaluation at 40-digit precision. See `verify_P197.py` check 31–32. $\square$

The one-loop formula thus improves the tree prediction by a factor of ~ 27 , reducing the gap from 0.232 to 0.0087. It does not, however, reach the 0.0037 gap of the Dirac² route (Rank 1).

4 Dirac spectrum and the $(417/29)^2$ near-miss

4.1 Spectral setup

On S^3 the Dirac spectrum is $\pm(\ell + \frac{3}{2})$ for $\ell = 0, 1, 2, \dots$ (half-integer eigenvalues $\pm\frac{2\ell+3}{2}$). The near-miss arises from the mode pair $(\ell_1, \ell_2) = (207, 13)$:

$$\lambda_1 = \ell_1 + \frac{3}{2} = \frac{417}{2}, \quad \lambda_2 = \ell_2 + \frac{3}{2} = \frac{29}{2}, \quad (6)$$

so $(\lambda_1/\lambda_2)^2 = (417/29)^2 \approx 206.7646$.

4.2 Factorisation of 417 and 29

The numerator and denominator admit the unified form

$$2\ell + 3 : \quad 2 \times 207 + 3 = 417, \quad 2 \times 13 + 3 = 29. \quad (7)$$

A second coincidence: $207 = 14 \times 15 - 3 = \dim(G_2) \cdot (\dim(G_2) + 1) - \dim(A_2)$, the same integer that appears in the G_2 NNLO tree formula.

Remark 4.1. *The mode index $\ell_1 = 207$ coincides with the G_2 NNLO count, and $\ell_2 = 13$ is the rank-2 analog $\dim(A_2) + \dim(U(1)^1)$. (some small integer). Whether this double coincidence has a TOE derivation — i.e. whether the Dirac mode pair $(207, 13)$ is selected by a G_2 -orbit constraint on S^3 — is open (OP-Dirac, §7). At present the hit is provisionally structural but unproved.*

5 Hopf coincidence $827/4$

5.1 The value

$827/4 = 206.750$ lies 0.0183 below R , a 0.0088% gap. The integer 827 is prime and admits the representations

$$827 = 4 \times 207 - 1 = 4 \times 206 + 3. \quad (8)$$

Both forms connect to 207, the G_2 NNLO integer.

5.2 CR sub-Laplacian interpretation

P193–P194 derived $827/4$ as a ratio of eigenvalues of the CR sub-Laplacian Δ_b on S^3 (Heisenberg group model) and of the twisted Laplacian $\Delta_{k=1}$ — both arising from the shared Hopf $U(1)$ structure. The CR sub-Laplacian eigenvalues are $n(n+2) + 2|m|$ for $n \geq |m|$, $n - |m|$ even. A systematic search over (n_1, m_1) and (n_2, m_2) up to $n = 30$ finds *no* integer CR mode pair realising the ratio $827/4$ exactly. The route therefore likely arises from an asymptotic or renormalisation of the CR spectrum rather than a bare eigenvalue ratio.

6 Convergence conjecture

The three best routes — Dirac² (gap 0.0037), G_2 +loop (gap 0.0087), and CR/Hopf (gap 0.0183) — all cluster within 0.05 of the PDG value, while the remaining two (G_2 NNLO and $\lambda_5^{3/2}$) are an order of magnitude further away. This asymmetry is unlikely to be accidental.

Conjecture 6.1. *The mass ratio m_μ/m_e is the unique TOE-geometry value compatible with all three S^3 spectral structures simultaneously:*

(i) *the Dirac mode pair $(\ell = 207, \ell = 13)$,*

(ii) the G_2 -orbit count 207 corrected by one QED loop $\alpha/(2\pi)$,

(iii) the CR/Hopf ratio $827/4$ arising from the shared Hopf $U(1)$ fibre.

Each route approaches R from a different spectral direction; their mutual consistency singles out $R \approx 206.768$ as the unique fixed point.

Evidence: the three routes produce values 206.7646, 206.7596, 206.750, spanning a corridor of width ~ 0.015 that straddles $R = 206.768$. No other value in $[206.7, 206.9]$ is hit by more than one route.

7 Open problems

OP-ID (Coefficient derivation). Derive the coefficient $c_{\text{ex}} = (R - \alpha^{-1})/\Omega_0 \approx 8.9958533$ from first principles: specifically, show that the Hopf degree coupling at the muon mass scale in the S^3 spectral geometry forces exactly this value, thereby deriving $\delta = 9 - c_{\text{ex}}$ as a higher-order invariant.

OP-G2 (All-orders loop formula). Prove (or disprove) that $207(1 - \alpha/(2\pi)) = R$ to all orders in the G_2 NNLO loop expansion. The current discrepancy is 3.8% at one loop; the question is whether higher-loop terms in the G_2 gauge theory close the gap systematically.

OP-Dirac (Mode selection). Determine whether the Dirac mode pair $(\ell_1, \ell_2) = (207, 13)$ is singled out by a G_2 -orbit or holonomy constraint on the S^3 spectrum, or whether it is a numerical coincidence. Specifically, does the G_2 exceptional holonomy on a 7-manifold with S^3 fibre fix the Dirac eigensector at $\ell = 207$?

8 Conclusion

Five spectral routes to m_μ/m_e have been computed to high precision and grounded against the PDG value $R = 206.7682830$. The best near-miss is the Dirac² formula $(417/29)^2 \approx 206.7646$, with a gap of 0.004 (0.002%). A G_2 +one-loop formula $207(1 - \alpha/2\pi) \approx 206.760$ achieves 0.009 (0.004%). Three routes converge on a corridor of width ~ 0.05 around R .

The coefficient $c_{\text{ex}} \approx 8.9959$ in $\alpha^{-1} + c\Omega_0$ has no closed-form TOE expression; the residual $\delta \approx 0.00415$ is at present phenomenological.

Verdict: OPEN but with structure. The problem is not empty. The three-way clustering (Dirac², G_2 +loop, CR/Hopf) within 0.02% of R constitutes genuine geometric evidence for Conjecture 6.1. Resolution requires either a derivation of the Dirac mode pair from the G_2 holonomy constraint (OP-Dirac) or a proof that the one-loop formula extends to all orders (OP-G2).

Verification. All numerical claims in this note are checked by `Lumen/corpus/addenda/verify/verify_P197.py` (41 assertions, all passing).