

Addendum 195: Running Coupling and Mass Ratio

Can $m_\mu/m_e \approx 206.77$ Emerge from TOE Renormalisation?

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Abstract

Addendum 191 proved no static ratio of S^3 eigenvalues equals $\mathcal{R} = m_\mu/m_e = 206.7683$ (CODATA 2018). This note exhausts four dynamical routes: (i) the $(8/3)^2 \times 29$ spectral near-miss, (ii) QED renormalisation-group running, (iii) GON lapse running over one breath period, and (iv) a PSLQ integer-relation scan over TOE constants. The G_2 /NNLO mechanism of P133 is summarised. **Verdict: OPEN.** Closest arithmetic approach: $\alpha_0^{-1} + 9\Omega_0 = \frac{25\pi^3}{4} + \pi^2 + \pi \approx 206.800$, error 0.016%, with canonically motivated spectral coefficients (25 and 9 are S^3 degeneracies at $\ell = 4$ and $\ell = 2$ respectively).

1 Introduction

The S^3 spectrum assigns eigenvalue $\lambda_\ell = \ell(\ell + 2)$ at level ℓ , degeneracy $(\ell + 1)^2$. P191 proved no ratio $\lambda_{\ell'}/\lambda_\ell$ equals $\mathcal{R}$. The TOE constants used throughout:

$$\alpha_0^{-1} = 4\pi^3 + \pi^2 + \pi \approx 137.0363, \quad \alpha = 1/\alpha_0^{-1} \approx 7.297 \times 10^{-3},$$

$$\Omega_0 = \pi^3/4 \approx 7.7516, \quad \mathcal{T} = \pi\alpha_0^{-1} \approx 430.51, \quad \mathcal{R} = 206.7683.$$

The static route is closed; this addendum investigates whether a *running/dynamical* mechanism produces $\mathcal{R}$.

2 The $(8/3)^2 \times 29$ Near-Miss

Spectral data. S^3 eigenvalues: $\lambda_1 = 1 \cdot 3 = 3$, $\lambda_2 = 2 \cdot 4 = 8$ (**C1**).

$$\frac{\lambda_2}{\lambda_1} = \frac{8}{3}, \quad \left(\frac{8}{3}\right)^2 = \frac{64}{9} \approx 7.1111. \quad (\text{C2})$$

Dividing the target:

$$\frac{\mathcal{R}}{64/9} = \frac{206.7683 \times 9}{64} = \frac{1861.01}{64} \approx 29.078. \quad (\text{C3})$$

Nearest integer $N = 29$ gives

$$\frac{64}{9} \times 29 = \frac{1856}{9} = 206.222, \quad \text{error} = \frac{206.768 - 206.222}{206.768} \approx 0.264\%. \quad (\text{C4})$$

S^3 harmonic degeneracies. Cumulative counts $\sum_{\ell=0}^L (\ell + 1)^2$: 1, 5, 14, 30, 55, ... (**C5**). Level $\ell = 2$: cumulative 14 = $\dim G_2$ (**C6**); level $\ell = 3$: cumulative 30 $\neq$ 29.

TOE integers near 29. $\dim J_3(\mathbb{O}) = 27$: gives $(8/3)^2 \times 27 = 192.0$ (error 7.1%). $h(E_8) = 30$: gives $(8/3)^2 \times 30 = 213.3$ (error 3.2%). Neither equals 29. Positive-root counts: $|\Phi^+(E_6)| = 36$, $|\Phi^+(F_4)| = 24$, $|\Phi^+(G_2)| = 6$. The integer 29 is the 10th prime; it is *not* a natural TOE integer.

Verdict. Near-miss at 0.264%; 29 structurally unmotivated. **OPEN:** if a TOE derivation of $N = 29$ is found, the 0.26% residual requires an NLO correction of order α .

3 Renormalisation Group Approach

One-loop QED running.

$$\ln \mathcal{R} = \ln 206.768 \approx 5.3316, \quad \ln(m_\mu^2/m_e^2) = 10.663, \quad (\text{C7})$$

$$\Delta(\alpha_0^{-1}) = \frac{1}{3\pi} \ln \frac{m_\mu^2}{m_e^2} = \frac{10.663}{9.4248} \approx 1.131. \quad (\text{C8})$$

Hence $\alpha_0^{-1}(m_\mu) = 137.036 - 1.131 = 135.905$ (C9) and

$$\frac{\alpha(m_\mu)}{\alpha(m_e)} = \frac{\alpha_0^{-1}(m_e)}{\alpha_0^{-1}(m_\mu)} = \frac{137.036}{135.905} \approx 1.0083. \quad (\text{C10})$$

The coupling changes by 0.83%—not by a factor 207.

Self-consistent RG requirement. Demanding $\alpha_0^{-1}(m_\mu) = \mathcal{R} \cdot \alpha_0^{-1}(m_e) = 28\,335$ would require a one-loop coefficient

$$b_0 = \frac{(\mathcal{R} - 1)\alpha_0^{-1}}{\ln \mathcal{R}} = \frac{205.77 \times 137.036}{5.332} \approx 5289. \quad (\text{C20})$$

No perturbative theory has $b_0 \sim 5000$. The kernel constitutional pin also forbids running: α is a fixed geometric constant.

Verdict. CLOSED. Standard QED running is a $\sim 1\%$ effect; a non-perturbative large- b_0 scenario is structurally forbidden.

4 GON Lapse Running

Setup. GON lapse $m(\beta) = |\sin 2\beta|$. Over one breath period $\mathcal{T}$, β advances by $\Delta\beta = \Omega_0 = \pi^3/4$, so

$$2\Delta\beta = \frac{\pi^3}{2} \approx 15.503 \text{ rad}. \quad (\text{C11})$$

Reducing mod 2π :

$$\varphi \equiv \frac{\pi^3}{2} \bmod 2\pi = 15.503 - 4\pi \approx 2.9368 \text{ rad} = 168.26^\circ. \quad (\text{C12})$$

Trig values: $\cos \varphi \approx -0.9791$ (C13), $\sin \varphi \approx 0.2034$ (C14).

Exact ratio equation. Setting $m(\beta_e)/m(\beta_\mu) = \mathcal{R}$ with $\theta = 2\beta_e$:

$$\frac{\sin \theta}{\sin(\theta + \varphi)} = \mathcal{R} \implies \tan \theta = \frac{\mathcal{R} \sin \varphi}{1 - \mathcal{R} \cos \varphi} = \frac{206.77 \times 0.2034}{1 + 206.77 \times 0.9791} \approx \frac{42.06}{203.4} \approx 0.2068, \quad (1)$$

giving $\beta_e \approx 0.1019 \text{ rad} = 5.84^\circ$ (C15). Verification: $|\sin 0.2038|/|\sin(0.2038 + 2.9368)| \approx 0.2024/9.79 \times 10^{-4} = 206.77 \checkmark$

Is β_e natural? $\beta_e/\alpha \approx 13.97$, close to $14 = \dim G_2$ but not exact.

Verdict. CLOSED (no predictive power). The mechanism produces $\mathcal{R}$ exactly for a tuned β_e . Without a first-principles derivation of β_e , no prediction is made.

5 PSLQ Scan over TOE Constants

We search for $a, b, c \in \mathbb{Z}$, $|\cdot| \leq 10$, expressing $\mathcal{R}$ as a linear combination of $\{\alpha_0^{-1}, \pi^2, \pi^3, \Omega_0, \pi\}$.

Explicit candidates (arithmetic shown).

$$\alpha_0^{-1} + 7\pi^2 = 137.036 + 7 \times 9.870 = 137.036 + 69.087 = 206.123, \quad \text{error } 0.312\%, \quad (\text{C16})$$

$$\alpha_0^{-1} + 7\pi^2 + 1 = 207.123, \quad \text{error } 0.172\%, \quad (\text{C17})$$

$$\alpha_0^{-1} + 9\Omega_0 = 137.036 + 9 \times 7.752 = 137.036 + 69.764 = 206.800, \quad \text{error } 0.0155\%. \quad (\text{C18})$$

Structure of the best hit (C19).

$$\alpha_0^{-1} + 9\Omega_0 = (4\pi^3 + \pi^2 + \pi) + \frac{9\pi^3}{4} = \frac{25\pi^3}{4} + \pi^2 + \pi \approx 206.800. \quad (2)$$

Coefficient $25 = (4 + 1)^2$ is the S^3 degeneracy at $\ell = 4$; coefficient $9 = (2 + 1)^2$ is the degeneracy at $\ell = 2$. Absolute error: $206.800 - 206.768 = 0.032$ (0.016%). A layer-fraction correction $\delta\Omega_0 \approx -0.0035$ of order α^2/π^2 would close the gap.

Verdict. PROMISING. No exact relation found, but $\alpha_0^{-1} + 9\Omega_0$ is the closest approach at 0.016% with canonical spectral coefficients. Pursue at two-loop in Addendum 196.

6 G_2 /NNLO Mechanism (P133 Summary)

G_2 geometry (C21): $\dim G_2 = 14$, $|\text{roots}| = 12$, long/short root ratio $= \sqrt{3}$, Weyl group order $= 12$, Coxeter number $h = 6$.

Orbit counting (C22).

$$\dim(G_2) \times (\dim(G_2) + 1) = 14 \times 15 = 210, \quad \text{error from } \mathcal{R} \approx 1.6\%. \quad (3)$$

P133 subtracts an $A_2 \subset G_2$ correction of 3 units (the three short-root pairs embedding in A_2 , with the long-root ratio $(\sqrt{3})^2 = 3$ entering the two-loop integrand):

$$14 \times 15 - 3 = 207, \quad \text{error from } \mathcal{R} \approx 0.11\%. \quad (4)$$

The exact NLO coupling correction to bridge $207 \rightarrow 206.768$ is of order $\alpha \approx 0.73\%$ — consistent in size with the gap.

Verdict. OPEN. The A_2 -subtraction step needs explicit verification against the P133 Casimir computation.

7 Summary Table and Verdict

Route	Closest value	Error	Status
$(8/3)^2 \times 29$	206.222	0.264%	OPEN (29 unmotivated)
QED RG running	α ratio ≈ 1.008	$\gg 1$	CLOSED
GON lapse (tuned β_e)	206.768 exact	0%	CLOSED (no prediction)
$\alpha_0^{-1} + 9\Omega_0 = \frac{25\pi^3}{4} + \pi^2 + \pi$	206.800	0.016%	PROMISING
G_2/NNLO (P133): $14 \times 15 - 3$	207	0.11%	OPEN (verify P133)

Overall verdict: OPEN. No mechanism closes $\mathcal{R}$ from first principles. Two routes remain viable: (1) *Spectral arithmetic* — $\alpha_0^{-1} + 9\Omega_0 \approx 206.800$ misses by 0.032; a two-loop layer-fraction correction of order α/π is plausible (Addendum 196). (2) $G_2/NNLO$ — integer 207 from P133 is 0.11% from $\mathcal{R}$; an NLO coupling factor closes the gap at the right order of magnitude.

The best numerical approach to $\mathcal{R}$ found in this addendum is

$$\boxed{\frac{25\pi^3}{4} + \pi^2 + \pi \approx 206.800, \quad \delta = 0.016\%}. \quad (5)$$

The GON lapse mechanism is noted as mathematically capable but physically vacuous without an independent derivation of $\beta_e \approx 0.1019$ rad.

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