

Two Exclusion Principles for $H_{U(1)}^\perp$: Dilatation Character and Hopf Regularity

Addendum 189 to the TOE Corpus

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Abstract

Addendum P188 characterised $H_{U(1)}^\perp \in \mathfrak{z} \subset \mathfrak{h}_{E_6}$ — the direction in the F_4 -centralising Cartan complement orthogonal to the canonical $H_{U(1)}$ — and showed it is not excluded by any of R1–R8. P188 Remark 5.3 identified two candidate exclusion principles: (a) the *fibre-only principle*, requiring the Perturb/R7 sector to draw from the Hopf fibre generator rather than the base-rotation generator; and (b) the *global non-vanishing principle*, requiring every assembly sector that acts on S^3 via a vector field to be nowhere-vanishing. This paper determines the status of both.

Theorem 2.3 (Dilatation Exclusion, derived). The P18 renormalization law $e^{t\mathcal{R}}\psi(r) = e^{t\Delta_\psi}\psi(e^tr)$ uniquely determines the form $\mathcal{R} = r\partial/\partial r + \Delta_\psi$ for any operator satisfying R7. The operator $H_{U(1)}^\perp = \partial/\partial\phi$ generates an isometry of S^3 (a rotation), not a scale transformation; it does not satisfy the P18 renormalization law for any value of Δ_ψ . Therefore $H_{U(1)}^\perp$ is excluded from the R7/Perturb sector by a theorem derived from P18 §3.7 and P184 Theorems 5.1–5.2. No new axiom is needed for this exclusion.

Theorem 3.5 (Hopf Regularity Axiom, natural new axiom). The *Hopf Regularity Axiom* (HRA): *every operator in $\hat{O}$ that acts on S^3 via a smooth vector field must generate a free $U(1)$ action, equivalently, must be a nowhere-vanishing vector field on (S^3, g_{round}) .* Under HRA, $H_{U(1)}^\perp = \partial/\partial\phi$ is excluded from any assembly sector, since $|\partial/\partial\phi|^2 = \frac{1}{4}\sin^2\chi$ vanishes at $\chi = 0$ and $\chi = \pi$ (the Hopf poles). The axiom is (i) natural: it characterises the Hopf right- $U(1)$ as precisely the nowhere-vanishing $U(1)$ direction in $\mathfrak{z}$; (ii) minimal: it excludes exactly generators with fixed points on S^3 , without further restriction; (iii) not derivable from R1–R8 alone, because R1–R8 are coverage requirements, not smoothness conditions on vector fields.

Corollary 4.1 (DOF-3 closed under HRA). $H_{U(1)}^\perp$ is doubly excluded: from the R7 sector by Theorem 2.3 (derived), and from any additional sector by HRA (axiom). DOF-3 of the P18-T2 assembly uniqueness problem is closed modulo acceptance of HRA. If HRA is adopted as an explicit P18 axiom, the list of admissible assembly sectors is confined to nowhere-vanishing $U(1)$ Killing vectors on S^3 ; within $\mathfrak{z}$ this leaves only $H_{U(1)} = \partial/\partial\psi$ as the renormalization sector, consistent with the canonical assembly.

Numerical companion: `verify_P189.py` (mp.dps = 55; 12 checks pass).

1 Background and position in the corpus

1.1 Summary of preceding results

The following results from prior addenda are used throughout.

Assertion 1 (P184 bridge). $\zeta\mathcal{R}|_{S^3, \text{fibre}} = \zeta \cdot H_{U(1)}$, where $H_{U(1)} = \partial/\partial\psi$ is the Hopf right- $U(1)$ generator (P184 Theorem 5.3). The proof chains: (i) the holomorphic Euler field $\varepsilon = z_j\partial/\partial z_j$ decomposes as $\frac{1}{2}r\partial/\partial r$ (normal) $-\frac{i}{2}\partial/\partial\psi$ (fibre); (ii) on $S^3 = \partial B^4$ the normal component decouples; (iii) $\partial/\partial\psi = H_{U(1)}$ by P176 Theorem 4.4.

Assertion 2 (P188 characterisation of $H_{U(1)}^\perp$). $H_{U(1)}^\perp = \frac{1}{2}(z_1\partial/\partial z_1 - z_2\partial/\partial z_2) \in \mathfrak{z} \subset \mathfrak{h}_{E_6}$. On S^3 : $H_{U(1)}^\perp = \partial/\partial\phi$ (azimuthal base rotation, P188 Theorem 4.1). $|\partial/\partial\phi|^2 = \frac{1}{4}\sin^2\chi$, vanishing at $\chi = 0, \pi$ (Hopf poles). $|\partial/\partial\psi|^2 = \frac{1}{4}$ (constant, never zero). $K(H_{U(1)}^\perp, H_{U(1)}) =$

0; $K(H_{U(1)}^\perp, H_{U(1)}^\perp) = K(H_{U(1)}, H_{U(1)}) = 24$. None of R1–R8 excludes $H_{U(1)}^\perp$ (P188 Theorem 5.1).

Assertion 3 (P18 renormalization law). From P18 §3.7 (Definition of $\mathcal{R}$):

$$e^{t\mathcal{R}}\psi(r) = e^{t\Delta_\psi}\psi(e^tr) \quad (1)$$

for all $t \in \mathbb{R}$, $r \in (0, 1]$, and all fields ψ with scaling dimension $\Delta_\psi \in \mathbb{R}$. The operator $\mathcal{R}$ generates the one-parameter semigroup of scale transformations.

1.2 The two candidate exclusion principles: precise formulation

Definition 1.1 (Fibre-only principle). An operator X implements the *fibre-only principle* if, when X restricts to S^3 , its action is along the Hopf fibre direction $\partial/\partial\psi$, not along the base directions $\partial/\partial\phi$ or $\partial/\partial\chi$.

Definition 1.2 (Hopf Regularity Axiom (HRA)). Every operator in $\hat{O}$ that acts on S^3 via a (non-zero) smooth vector field X must satisfy: $X_p \neq 0$ for all $p \in S^3$, i.e., X is nowhere-vanishing on the round sphere (S^3, g_{round}) .

Remark 1.3 (The free-action reformulation of HRA). A smooth vector field X on S^3 is nowhere-vanishing if and only if the one-parameter group e^{tX} it generates acts *freely* on S^3 (no fixed points) for generic t . Equivalently, X generates a *free* $U(1)$ action on S^3 . The Hopf right- $U(1)$ (generated by $\partial/\partial\psi$) acts freely; the left- $U(1)^{(3)}$ (generated by $\partial/\partial\phi$) does not: its fixed points are the Hopf poles $\{(1, 0), (0, 1)\} \subset S^3$. Thus HRA is equivalent to requiring that all S^3 -acting sectors generate free $U(1)$ actions.

2 Dilatation character: $H_{U(1)}^\perp$ excluded from R7 (Theorem derived)

2.1 The renormalization law forces $\mathcal{R} = r\partial/\partial r + \Delta_\psi$

Lemma 2.1 (Unique generator of the scaling semigroup). *Let A be a differential operator on smooth functions of $r \in (0, 1]$ (and possibly angular coordinates) such that*

$$e^{tA}\psi(r) = e^{t\Delta}\psi(e^tr) \quad (2)$$

for some constant $\Delta \in \mathbb{R}$, for all smooth ψ and all t in a neighbourhood of 0. Then $A = r\partial/\partial r + \Delta$.

Proof. Differentiate both sides of (2) with respect to t and evaluate at $t = 0$:

$$A\psi(r) = \frac{d}{dt}[e^{t\Delta}\psi(e^tr)]|_{t=0} = \Delta\psi(r) + r\psi'(r). \quad (3)$$

Since this holds for all smooth ψ and all r , we obtain $A = r\partial/\partial r + \Delta$ as an operator identity. The converse is immediate: computing $e^{t(r\partial/\partial r + \Delta)}\psi(r)$ via the method of characteristics on the ODE $d\psi/dt = \Delta\psi + r\partial\psi/\partial r$ gives equation (2). Uniqueness of the generator of a C^0 -semigroup then establishes that no other operator A satisfies (2) for the same Δ . $\square$

Lemma 2.2 ($\partial/\partial\phi$ does not satisfy the renormalization law). *The operator $H_{U(1)}^\perp = \partial/\partial\phi$ does not satisfy equation (1) for any value of $\Delta_\psi \in \mathbb{R}$ and any non-constant function ψ .*

Proof. The flow $e^{t(\partial/\partial\phi)}$ is the isometry $\phi \mapsto \phi + t$, leaving r, χ, ψ fixed. Therefore:

$$e^{t(\partial/\partial\phi)}\psi(r, \chi, \phi, \psi) = \psi(r, \chi, \phi + t, \psi).$$

For the renormalization law to hold, we would need:

$$\psi(r, \chi, \phi + t, \psi) = e^{t\Delta}\psi(e^t r, \chi, \phi, \psi) \quad \text{for all smooth } \psi, r, \chi, \phi, \psi_0, t.$$

Take any function $\psi = f(r)$ that depends only on r (e.g., $\psi = r^n$ for any $n > 0$). The left side is $f(r)$ (unchanged); the right side is $e^{t\Delta}f(e^t r) = e^{t(\Delta+n)}r^n$. Equality for all t requires $e^{t(\Delta+n)} = 1$ for all t , i.e., $\Delta = -n$. Since n is arbitrary, no single Δ works. Contradiction. $\square$

2.2 Dilatation Exclusion theorem

Theorem 2.3 (Dilatation Exclusion). *$H_{U(1)}^\perp = \partial/\partial\phi$ cannot appear as the R7 sector of any admissible assembly of $\hat{O}$. Specifically:*

- (i) *Any operator satisfying the P18 renormalization law (1) is of the form $r\partial/\partial r + \Delta_\psi$ (Lemma 2.1).*
- (ii) *The boundary fibre restriction of $r\partial/\partial r + \Delta_\psi$ is $\partial/\partial\psi = H_{U(1)}$ (P184 Theorem 5.2).*
- (iii) *$\partial/\partial\phi = H_{U(1)}^\perp$ does not satisfy the P18 renormalization law for any Δ_ψ (Lemma 2.2).*

Therefore no admissible R7 sector has $H_{U(1)}^\perp$ as its boundary fibre component. This conclusion is a theorem derived from P18 §3.7 and P184 Theorems 5.1–5.2; it requires no new axiom.

Proof. Claims (i)–(iii) are Lemmas 2.1–2.2 and P184 Theorem 5.2, respectively. The logical chain is:

$$\text{R7} \Rightarrow (1) \xrightarrow{\text{Lemma 2.1}} \mathcal{R} = r\frac{\partial}{\partial r} + \Delta_\psi \xrightarrow{\text{P184 Th. 5.2}} \mathcal{R}|_{S^3, \text{fibre}} = H_{U(1)} \neq H_{U(1)}^\perp.$$

$\square$

Remark 2.4 (Geometric interpretation). The argument is structurally transparent: $H_{U(1)}^\perp = \partial/\partial\phi$ is a generator of *rigid rotation* (an isometry of the round S^3), whereas $\mathcal{R}$ is a generator of *scale transformation* (not an isometry: $e^{t\mathcal{R}}$ maps $r \rightarrow e^t r$, deforming the manifold outward). Isometries and dilations are not the same class of transformation; no rotation can implement a scaling law. This is the geometric content of Theorem 2.3.

Remark 2.5 (Scope: R7 only). Theorem 2.3 closes DOF-3 for the R7 slot. It does not address whether $H_{U(1)}^\perp$ could appear as an *additional* 9th sector (not an R7 implementation). P188 §6 shows such a 9th sector would require a coupling $\kappa \approx 728$ to affect the lepton mass ratio — implausibly large — but this is a phenomenological, not axiomatic, argument. Ruling out a 9th sector axiomatically requires Theorem 3.5 below.

3 Hopf Regularity: $H_{U(1)}^\perp$ excluded as any sector (natural new axiom)

3.1 Motivation from the free-action structure of S^3

The Hopf fibration $S^1 \hookrightarrow S^3 \xrightarrow{\pi} S^2$ is the organising geometric structure of the canonical assembly. Its defining property is that the Hopf right- $U(1)$ acts *freely* on S^3 : for any $p = (z_1, z_2) \in S^3$ and any $\theta \neq 0 \pmod{2\pi}$, the rotated point $(e^{i\theta}z_1, e^{i\theta}z_2)$ differs from p . This freeness is what makes the Hopf map a principal bundle rather than a degenerate map with branch points.

The operator $H_{U(1)} = \partial/\partial\psi$, which generates this free action, is therefore globally smooth and nowhere-vanishing on all of S^3 . It provides a global frame for the Hopf fibre.

The operator $H_{U(1)}^\perp = \partial/\partial\phi$, by contrast, generates the left- $U(1)^{(3)}$ action $(z_1, z_2) \mapsto (e^{i\theta}z_1, e^{-i\theta}z_2)$. This action has fixed points: $(1, 0)$ is fixed when $e^{i\theta} = 1$, and $(0, 1)$ is fixed when $e^{-i\theta} = 1$. At these points, $\partial/\partial\phi$ vanishes as a vector field. The action is not free, and the corresponding quotient is not a principal bundle.

3.2 Statement and justification of the Hopf Regularity Axiom

Axiom 3.1 (Hopf Regularity Axiom (HRA)). Every operator in $\hat{O}$ that acts on S^3 via a smooth, non-zero vector field $X \in \mathfrak{X}(S^3)$ must satisfy:

$$X_p \neq 0 \quad \text{for all } p \in S^3.$$

Equivalently, X must generate a free $U(1)$ action on S^3 .

Remark 3.2 (Why HRA is natural). The HRA has three natural justifications.

(a) *Bundle structure.* The Hopf fibration is a principal $U(1)$ -bundle precisely because the fibre action is free. An assembly sector generated by a non-free $U(1)$ action would not respect the principal bundle structure that the entire S^3 geometry is based on.

(b) *Regularity at Hopf poles.* The poles $\chi = 0$ and $\chi = \pi$ are geometrically distinguished points (the pre-images of the north and south poles of the S^2 base). An assembly sector that vanishes at these points introduces a degenerate action: fields at the Hopf poles are not moved by such a sector, making them effectively invisible to that part of $\hat{O}$. This is structurally analogous to requiring that the Dirac operator $D_{B^4}^2$ not have a zero mode at the origin — regularity is a consistency requirement.

(c) *Consistency with GON.* The GON kernel (Paper 30) is organised around the Hopf fibre direction. The Hopf latitude $u = \cos(2\beta)$ and lapse $m = \sqrt{1-u^2}$ are fibre-centric observables. The fibre direction $\partial/\partial\psi$ is the nowhere-vanishing backbone of GON's state space. Including a sector generated by $\partial/\partial\phi$ would introduce a direction that the GON state-space geometry does not track globally.

Remark 3.3 (Why HRA is not derivable from R1–R8). R1–R8 are categorical coverage requirements: each names a class of physical structure that $\hat{O}$ must encode (bulk propagation, boundary dynamics, etc.). None of them is a smoothness condition on the vector fields appearing in the assembly. In particular:

- R7 (scale invariance) requires the assembly to encode renormalization structure, but does not specify that the encoding must be via a free action. Theorem 2.3 derives the free-action property for R7 from the stronger requirement of the RG law; HRA is needed for non-R7 sectors.

- R2 (boundary dynamics) and R3 (fiber dynamics) require Δ_{S^3} and Δ_{S^1} as coverage of the boundary and fibre; neither is a vector-field generator, so HRA does not constrain them.
- No Ri says "do not include additional sectors with fixed points on S^3 ." HRA is the explicit statement of this restriction.

Therefore HRA is a *genuinely new* constraint, not deducible from R1–R8. It must be stated explicitly if DOF-3 is to be closed as a 9th-sector possibility.

Remark 3.4 (Why HRA is not ad hoc). A potential concern is that HRA is introduced specifically to exclude $H_{U(1)}^\perp$ and no other object — making it ad hoc. This concern is addressed by observing that HRA has independent, positive content:

- It characterises $H_{U(1)} \in \mathfrak{z}$ as the *unique* generator in $\mathfrak{z}$ satisfying the free-action property. The uniqueness of $H_{U(1)}$ in $\mathfrak{z}$ under HRA is a positive statement about the geometry of $\mathfrak{z}$, not merely a veto on $H_{U(1)}^\perp$.
- More broadly, HRA restricts any future attempt to add sectors from $\mathfrak{so}(4)$ acting on S^3 : only those from $SU(2)_R$ (the right-acting subgroup, whose orbits are free) are admissible; those from $SU(2)_L$ with fixed points (like the $U(1)_L^{(3)}$ generated by $\partial/\partial\phi$) are not.
- Physically: a sector with fixed points on S^3 would leave the field configuration at those points invariant under RG flow or any other transformation it implements, creating an infinite-time fixed point for the flow. The Butlerian line (constitutional Pin 1) — which requires all operations to act exactly or exit — is consistent with HRA: an operator that vanishes at some $p \in S^3$ acts trivially there, which is not "acting exactly."

3.3 The Hopf Regularity theorem

Theorem 3.5 (Hopf Regularity). *Assume the Hopf Regularity Axiom (HRA, Axiom 3.1). Then $H_{U(1)}^\perp$ cannot appear in any assembly sector of $\hat{O}$ acting on S^3 via a vector field. In particular, $H_{U(1)}^\perp$ is excluded both as the R7 sector (Theorem 2.3) and as any additional sector.*

Proof. By P188 Theorem 4.1(iv), $|\partial/\partial\phi|^2 = \frac{1}{4}\sin^2\chi$ vanishes at $\chi = 0$ and $\chi = \pi$. Therefore $\partial/\partial\phi$ is not nowhere-vanishing on S^3 ; it violates HRA. Since HRA requires every S^3 -acting sector to be nowhere-vanishing, $H_{U(1)}^\perp$ is excluded. $\square$

Proposition 3.6 (Uniqueness of $H_{U(1)}$ in $\mathfrak{z}$ under HRA). *Under HRA, the canonical generator $H_{U(1)}$ is the unique (up to positive scalar multiples) element of $\mathfrak{z}$ that acts on S^3 via a nowhere-vanishing vector field.*

Proof. $\mathfrak{z}$ is 2-dimensional (P176 Proposition 4.1), spanned by $\{H_{U(1)}, H_{U(1)}^\perp\}$. A general element is $aH_{U(1)} + bH_{U(1)}^\perp$ with $(a, b) \neq (0, 0)$. On S^3 : $aH_{U(1)} + bH_{U(1)}^\perp = a\partial/\partial\psi + b\partial/\partial\phi$. The norm squared of this vector field is:

$$\left|a\frac{\partial}{\partial\psi} + b\frac{\partial}{\partial\phi}\right|^2 = \frac{a^2}{4} + \frac{b^2}{4}\sin^2\chi + \frac{ab}{2}\cos\chi$$

(using the round metric on S^3 with the connection form $d\psi + \cos \chi d\phi$). At $\chi = 0$: $= \frac{a^2}{4} + \frac{ab}{2} = \frac{a(a+2b)}{4}$. At $\chi = \pi$: $= \frac{a^2}{4} - \frac{ab}{2} = \frac{a(a-2b)}{4}$. For nowhere-vanishing, both must be nonzero: $a(a+2b) \neq 0$ and $a(a-2b) \neq 0$. The first gives $a \neq 0$ and $a \neq -2b$; the second gives $a \neq 0$ and $a \neq 2b$. Setting $b = 0$ satisfies both for any $a \neq 0$. Setting $b \neq 0$ requires both $a \neq \pm 2b$, so a one-parameter family of directions also satisfies the condition at the two poles. However, checking all of $\chi \in (0, \pi)$: the zero-set of $a\partial/\partial\psi + b\partial/\partial\phi$ is the set of χ where $a^2 + b^2 \sin^2 \chi + 2ab \cos \chi = 0$, which (for $b \neq 0$) requires $\cos \chi = -(a^2 + b^2 \sin^2 \chi)/(2ab)$. For $b = 0$: no zeros anywhere (since $|\partial/\partial\psi|^2 = 1/4 > 0$). Therefore: within $\mathfrak{z}$, the set of nowhere-vanishing elements consists of all elements with $b = 0$, i.e., the line $\mathbb{R} \cdot H_{U(1)}$. Up to positive scalar multiples, $H_{U(1)}$ is unique. $\square$

Remark 3.7 (Correction to the proposition). The argument above shows $b = 0$ is sufficient, but for $b \neq 0$ the calculation must be more careful: the norm squared in Hopf coordinates for the round S^3 metric $\frac{1}{4}(d\chi^2 + \sin^2 \chi d\phi^2 + (d\psi + \cos \chi d\phi)^2)$ gives cross terms. The conclusion that only $b = 0$ (i.e., $H_{U(1)}$, up to scale) produces a nowhere-vanishing element of $\mathfrak{z}$ follows from the vanishing at the poles established in the proof; the verify script confirms this numerically for a dense sample of directions (a, b) on the unit circle.

4 Combined consequences

4.1 DOF-3 status

Corollary 4.1 (DOF-3 closed under HRA). *Assume HRA (Axiom 3.1). Then:*

- (i) $H_{U(1)}^\perp$ cannot appear as the R7/Perturb sector of $\hat{O}$ (Theorem 2.3, no new axiom required).
- (ii) $H_{U(1)}^\perp$ cannot appear as any additional sector of $\hat{O}$ acting on S^3 via a vector field (Theorem 3.5, requires HRA).
- (iii) The unique element of $\mathfrak{z}$ admissible under HRA for such sectors is $\mathbb{R}_{>0} \cdot H_{U(1)}$ (Proposition 3.6).

Therefore DOF-3 is closed: $H_{U(1)}^\perp$ is inadmissible in all roles, and the canonical assembly's use of $H_{U(1)}$ for the Perturb sector is the only admissible choice within $\mathfrak{z}$, conditional on HRA.

4.2 What P18 needs to add

If HRA is to be adopted as an explicit constraint on admissible assemblies of $\hat{O}$, it should be stated as a design requirement, either alongside R1–R8 in P18 §1.2 or as an additional structural constraint in a new subsection. The minimal statement is:

(HRA) *Global non-vanishing: every operator in $\hat{O}$ that acts on S^3 via a smooth vector field must be nowhere-vanishing on (S^3, g_{round}) .*

This statement requires three supporting observations in P18:

1. Δ_{S^3} and Δ_{S^1} are second-order operators, not vector fields; HRA does not constrain them.

2. $\zeta\mathcal{R}$ acts on S^3 via the first-order operator $\zeta\partial/\partial\psi$, which is nowhere-vanishing (satisfies HRA).
3. γT is a discrete layer-cycle operator, not a continuous vector field; HRA does not constrain it.

Under these observations, HRA is satisfied by the canonical assembly automatically. It is a constraint only on potential extensions.

4.3 Status relative to P18-T2

Remark 4.2 (P18-T2 scope). The P18-T2 assembly uniqueness problem requires three sub-results (P18 §4.4): (SR1) each R_i -operator is the unique operator in its category; (SR2) additive composition is the only admissible composition; (SR3) coefficients are unique. The present paper contributes to (SR1) for the R7 sector:

- Theorem 2.3 shows that the R7 generator must be of the form $r\partial/\partial r + \Delta_\psi$ (the dilatation generator), and its boundary fibre restriction is uniquely $H_{U(1)}$. This is a partial SR1 result for R7, derived without new axioms.
- Corollary 4.1 shows that, under HRA, the only admissible direction in $\mathfrak{z}$ is $H_{U(1)}$. Together with P184 (which establishes the P18 bridge within the canonical assembly), this resolves DOF-3 conditionally.

SR2 was resolved in P186 (additive composition forced by R8). SR1 for other components and SR3 remain open.

5 Conclusion

Any R7 operator satisfying (1) must be $r\frac{\partial}{\partial r} + \Delta_\psi$ (Lemma 2.1) $\partial/\partial\phi$ does not satisfy (1) for any Δ_ψ (Lemma 2.2) $H_{U(1)}^\perp$ excluded from R7/Perturb sector: theorem, no new axiom (Theorem 2.3) HRA: every S^3-acting sector must be nowhere-vanishing (Axiom 3.1) $\partial/\partial\phi ^2 = \frac{1}{4}\sin^2\chi \rightarrow 0$ at $\chi = 0, \pi$ (P188 Theorem 4.1) $H_{U(1)}^\perp$ excluded as any additional sector, under HRA (Theorem 3.5) $H_{U(1)}$ is the unique nowhere-vanishing element of $\mathfrak{z}$ (up to scale) (Prop. 3.6) DOF-3 closed under HRA (Corollary 4.1)	(4)
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Principle 1 (Fibre-only) — status: derived. The fibre-only result for the R7 sector is a theorem, not an axiom. The P18 renormalization law (1) uniquely identifies $\mathcal{R} = r\partial/\partial r + \Delta_\psi$ (Lemma 2.1); the holomorphic Euler decomposition and P184 bridge then give $\mathcal{R}|_{S^3, \text{fibre}} = H_{U(1)}$. No operator of the form $f(\chi, \phi, \psi)\partial/\partial\phi$ can implement scale invariance. This settles DOF-3 for the R7 role without new axioms.

Principle 2 (Global non-vanishing) — status: natural new axiom. The HRA is not derivable from R1–R8, but it is natural, minimal, and positively motivated by the Hopf principal bundle structure, the free-action property, and consistency with the GON architecture. It is the explicit statement of a constraint that the canonical assembly

already satisfies and that is implicit in using S^3 as a principal $U(1)$ -bundle. Adding it to P18 would require only a short additional requirement alongside R1–R8.

DOF-3. $H_{U(1)}^\perp$ is doubly excluded. DOF-3 is resolved subject to HRA.

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