

SR2 Closure: R8 Forces Additive Composition of the Five $\hat{O}$ Sectors; Kleisli Composition is Inadmissible

Addendum 186 to the TOE Corpus

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Contents

1	Background and Position in the Corpus	2
1.1	The P18-T2 assembly-uniqueness question	2
1.2	SR2: the composition question	3
1.3	Paper 34's Kleisli machinery	3
2	R8 Precisely: Moment Hierarchy Linearity	4
2.1	The geometric moments μ_n	4
2.2	The moment operator $\mathcal{M}$	4
2.3	Moment-linearity: the spectral requirement of R8	4
3	Kleisli Composition of Sectors: Definition and Cases	5
3.1	Kleisli assembly of operator sectors	5
3.2	Two cases	5
4	The SR2 Theorem	6
4.1	Additive assembly satisfies R8	6
4.2	Kleisli assembly violates R8	6
4.3	SR2 closure corollary	8
5	Scope and Remaining Open Sub-results	9
5.1	What SR2 closure establishes	9
5.2	Remaining open sub-results	9
5.3	Upgrade path if SR1 and SR3 are closed	10
6	Conclusion	10

Abstract

Paper 18’s assembly-uniqueness open problem (P18-T2, §4.4) is that the design requirements (R1)–(R8) may admit alternative admissible assemblies of the master operator $\hat{O}$ beyond the canonical additive form. A proof of uniqueness requires three independent sub-results: SR1 (each R_i ’s operator is the unique one in its category), SR2 (additive composition is the only admissible composition), and SR3 (the four coefficients are the only spectrum-matching assignment). Paper 34 identifies Kleisli composition — via its monad $(S^3, \text{return}, \triangleright=)$ — as the leading alternative to additive composition, making SR2 the tractable sub-result.

This paper closes SR2. We show that requirement R8 (moment hierarchy: connect spectrum to μ_n) forces additive composition and rules out Kleisli sequential composition in two cases that jointly exhaust all possibilities.

Case A (commuting sectors, Proposition 4.1 and Theorem 4.3(A)): Under additive assembly, the moment operator $M = \sum_n (\mu_n/\mu_0) P_n$ contributes an additive term $\beta(\mu_n/\mu_0)$ to eigenvalue E_n with a *uniform* coefficient β for all n . Under Kleisli (sequential product) composition, the effective coupling to μ_n is βG_n , where G_n is the product of all other sectors’ eigenvalues at mode n . Since G_n varies with n , the coupling is mode-dependent, violating R8’s uniform linearity requirement.

Case B (non-commuting sectors, Theorem 4.3(B)): The sectors of $\hat{O}$ are mutually non-commuting: $[D_{B^4}^2, \gamma T] \neq 0$ and $[\zeta \mathcal{R}, \Delta_{S^3}] \neq 0$. Kleisli sequential composition of non-commuting sectors produces mode-mixing, so the eigenstates of the Kleisli assembly are not the radial modes $\{\psi_n\}$ that diagonalise M . The mode-moment correspondence $\psi_n \leftrightarrow \mu_n$ required by R8 is destroyed.

Corollary (SR2 Closure, Corollary 4.6): R8’s moment hierarchy linearity is structurally incompatible with Kleisli composition of the $\hat{O}$ sectors. Additive composition is the only admissible composition under R8. SR1 and SR3 remain open.

Numerical companion: `verify/verify_P186.py` (`mp.dps = 55`; all 12 checks pass).

1 Background and Position in the Corpus

1.1 The P18-T2 assembly-uniqueness question

Paper 18 [1] assembles the master operator

$$\hat{O} = D_{B^4}^2 + \Delta_{S^3} + \Delta_{S^1} + V_{\text{self}}(r) + \lambda \rho(r) + \gamma T + \zeta \mathcal{R} + \beta \mathcal{M}, \quad (1)$$

where each sector is derived in a prior paper and the four coefficients $(\alpha, \gamma, \zeta, \beta)$ are pinned by geometric consistency. P18 §4.4 records as an open problem (P18-T2) whether the design requirements (R1)–(R8) uniquely determine this assembly:

Assertion 1 (P18 §4.4 open problem, verbatim). *Whether the design requirements (R1)–(R8) admit alternative admissible assemblies of the operator $\hat{O}$ — different orderings, coefficient assignments, additional sectors, or operadic compositions in place of additive composition — is open.*

A proof of P18-T2 uniqueness would require three independent sub-results [5]:

SR1. Each R_i ’s named operator is the unique operator in its category (bulk dynamics $\rightarrow D_{B^4}^2$, boundary $\rightarrow \Delta_{S^3}$, etc.).

SR2. Additive composition is the only admissible composition of the sectors — alternatives such as Kleisli composition (via Paper 34 [3]) or operadic composition (on S^3) are ruled out.

SR3. The coefficients $(\alpha, \gamma, \zeta, \beta)$ are the only spectrum-matching assignment given the Hopf-induced eigenvalue structure.

Addendum P184 [4] formally derived the $\text{Wheel} \leftrightarrow \hat{O}$ dictionary for the canonical assembly, noting that P18-T2 uniqueness is separate and not resolved. The present paper addresses SR2 specifically.

1.2 SR2: the composition question

SR2 asks whether additive composition ($\hat{O} = \sum_i K_i$) is the only admissible way to combine the sectors, or whether Kleisli composition — in which sectors are sequentially applied as Kleisli morphisms in a monad — provides an admissible alternative. Additive composition is the canonical choice of P18; Paper 34 supplies the Kleisli alternative.

The focus on Kleisli composition is motivated by the P18-T2 reframing memo [5], which names “composition via Kleisli arrows on a monad over the $\hat{O}$ -sector algebra, per Paper 34” as the leading operadic alternative, and by Paper 34 itself, which establishes the formal monad $(S^3, \text{return}, \triangleright=)$ as a compositional framework for LumenOS computations.

Assertion 2 (SR2 is tractable). SR2 is identified in [5] as the tractable sub-result because: (i) additive vs. Kleisli composition is a concrete algebraic distinction amenable to spectral analysis, and (ii) Paper 34’s Kleisli machinery is already formalized, providing the precise alternative to test against R8. SR1 and SR3 require additional representation-theoretic and spectral-rigidity machinery not yet available in the corpus.

1.3 Paper 34’s Kleisli machinery

We recall the essential constructions from Paper 34 [3].

Assertion 3 (P34 monad (recap)). Paper 34 defines the monad (T, η, μ) over S^3 :

- Carrier: $T(S^3) = \{(q_i, w_i)_i \mid q_i \in S^3, w_i \geq 0, \sum_i w_i \leq 1\}$ (scored distributions over S^3).
- Unit: $\eta(q) = \delta_q$ (point distribution at q).
- Bind: $m \triangleright= f = \eta(\bigcup_i w_i \cdot f(q_i))$ (weighted-merge and geodesic-centroid collapse).

The monad laws (left identity, right identity, associativity) are verified in Paper 34 Theorems 3.1–3.3.

Assertion 4 (P34 Kleisli category (recap)). The Kleisli category $\mathbf{Kl}(T)$ has:

- Objects: $q \in S^3$.
- Morphisms $q_A \rightarrow q_B$: attention steps $f : S^3 \rightarrow T(S^3)$.
- Kleisli composition: $(g \circ_K f)(q) = f(q) \triangleright= g$.

In code, Kleisli composition is the `depends_on` sequencing of plan steps: if step B depends on step A , execution is $f_A(q) \triangleright= f_B$ (Paper 34 Proposition 4.1).

The extension of Paper 34’s Kleisli framework to operator sectors of $\hat{O}$ is the candidate alternative specified in Assertion 2: each sector K_i is treated as a Kleisli morphism (a map from a system state to a scored distribution over states), and sectors are composed via $\circ_K$ rather than summed. We formalise this in Definition 3.1 below.

2 R8 Precisely: Moment Hierarchy Linearity

2.1 The geometric moments μ_n

The geometric density is $\rho(x) = 16\pi^3 x^3 + 3\pi^2 x^2 + 2\pi x$ (Paper 03 [2]). The n -th geometric moment is

$$\mu_n = \int_0^1 x^n \rho(x) dx = \frac{16\pi^3}{n+4} + \frac{3\pi^2}{n+3} + \frac{2\pi}{n+2}, \quad n \in \mathbb{N}_0. \quad (2)$$

Assertion 5 (Ground moment = α^{-1}). At $n = 0$: $\mu_0 = 4\pi^3 + \pi^2 + \pi = \alpha^{-1}$ (Paper 02–03 [2]). Numerically $\mu_0 \approx 137.036$.

Assertion 6 (Moment sequence is decreasing). Since $\rho(x) \geq 0$ on $[0, 1]$ and $x \in [0, 1]$, the factor x^n is pointwise non-increasing in n . Hence $\mu_0 > \mu_1 > \mu_2 > \dots > 0$. Numerically: $\mu_0 \approx 137.04$, $\mu_1 \approx 108.7$, $\mu_2 \approx 90.2$, $\mu_3 \approx 76.6$, $\mu_4 \approx 66.3$ (verified in `verify_P186.py`, Checks 1–2).

2.2 The moment operator $\mathcal{M}$

Let $\mathcal{H}$ be the Hilbert space of L^2 -sections of the spinor bundle over B^4 (P18 §2.3). The radial decomposition $\psi(r, \theta) = \sum_{n,l,m} R_{nl}(r) Y_{lm}(\theta) \chi_s$ gives a complete orthonormal system $\{\psi_{n,l,m}\}$. We write $\{\psi_n\}$ for the $l = 0, m = 0$ sector (s-wave modes), which diagonalise the radial part of $D_{B^4}^2$.

Assertion 7 (Moment operator from P18). From P18 §3.8: the moment operator is

$$\mathcal{M} = \sum_{n=0}^{\infty} \frac{\mu_n}{\mu_0} P_n, \quad (3)$$

where $P_n = |\psi_n\rangle\langle\psi_n|$ is the orthogonal projection onto the n -th radial mode. Eigenvalue of $\mathcal{M}$ on ψ_n : $\mu_n/\mu_0 = m_n$ (the normalised n -th moment). The operator $\mathcal{M}$ is diagonal in the mode basis, bounded, and positive.

Remark 2.1 (Scaled notation). We write $m_n = \mu_n/\mu_0$ for the normalised moments, so $m_0 = 1$, $m_1 \approx 0.793$, $m_2 \approx 0.658$, etc. The contribution of $\beta\mathcal{M}$ to the eigenvalue at mode n is βm_n .

2.3 Moment-linearity: the spectral requirement of R8

Assertion 8 (R8 from P18 §1.2). From P18 §1.2: requirement R8 is “*Moment hierarchy: Connect spectrum to μ_n .*” The named operator implementing R8 is $\beta\mathcal{M}$ with $\beta = 3\pi/20$ (P18 Proposition 4.2 [1]).

The word “connect spectrum to μ_n ” admits several interpretations. We formalise the one that follows from the P18 construction:

Definition 2.2 (R8 moment-linearity). An assembled operator $\hat{O}_{\text{asm}}$ satisfies *R8 moment-linearity* if there exists a universal constant $\beta \neq 0$ such that, for every radial mode $n \geq 0$,

$$\langle \psi_n | \hat{O}_{\text{asm}} | \psi_n \rangle = a_n + \beta m_n, \quad (4)$$

where a_n is the contribution from all sectors other than $\beta\mathcal{M}$, and the coupling coefficient β is the *same* for all n .

Remark 2.3 (Why uniformity of β is the key). Definition 2.2 captures the P18 construction precisely: the moment operator $\mathcal{M} = \sum_n m_n P_n$ contributes an additive correction βm_n to E_n with the same proportionality constant β for every mode. This is the *linear* coupling that P18 names “moment hierarchy”: the spectral ladder $\{E_n\}$ reflects the moment ladder $\{\mu_n\}$ through a fixed linear coefficient. An assembly that modulates β by a mode-dependent factor G_n satisfies a different (nonlinear) coupling.

3 Kleisli Composition of Sectors: Definition and Cases

3.1 Kleisli assembly of operator sectors

The five Wheel-mapped sectors of $\hat{O}$ are (CLAUDE.md Pin 5 [6]):

$$\text{Fork} \leftrightarrow D_{B^4}^2, \quad \text{Weld} \leftrightarrow \Delta_{S^3}, \quad \text{Plateau} \leftrightarrow \Delta_{S^1}, \quad \text{Oscillate} \leftrightarrow \gamma T, \quad \text{Perturb} \leftrightarrow \zeta \mathcal{R}. \quad (5)$$

Together with $\beta\mathcal{M}$ (R8), V_{self} , and $\lambda\rho$, these eight objects are the sectors of $\hat{O}$. In the canonical additive assembly they are summed. A Kleisli assembly sequences them as morphisms.

Definition 3.1 (Kleisli assembly of $\hat{O}$ sectors). A *Kleisli assembly* of the $\hat{O}$ sectors is an operator of the form

$$\hat{O}_K = K_{i_N} \circ K_{i_{N-1}} \circ \cdots \circ K_{i_1}, \quad (6)$$

where $\{K_{i_j}\}$ is an ordered selection of sectors (with scalar multiples as in the canonical assembly), and $\circ$ denotes sequential composition of linear operators on $\mathcal{H}$ — the operator-algebra realisation of Kleisli composition from Paper 34 Definition 4.1. In this realisation, $K_{j+1} \circ K_j$ is the operator $f_{j+1} \circ f_j$ in the Kleisli category $\mathbf{Kl}(T)$, with each K_i acting as a Kleisli morphism that maps a system state to a distribution over subsequent states.

Remark 3.2 (The operator-algebra realisation). Paper 34 §4 defines Kleisli composition as $(g \circ_K f)(q) = f(q) \triangleright g$. In the operator-algebra context, where f and g are linear maps on $\mathcal{H}$, this reduces to sequential application: the output of f becomes the input of g , i.e., $g \circ f$ in the usual sense of operator composition. Definition 3.1 adopts this realisation, which is the natural extension of Paper 34’s computational Kleisli composition to the operator-algebra setting referenced in Assertion 2.

3.2 Two cases

We distinguish two cases based on whether the sectors in the Kleisli chain commute:

Definition 3.3 (Case A and Case B). • **Case A** (commuting): all sectors in the Kleisli chain commute with $\mathcal{M}$ and with each other. In this case the radial modes $\{\psi_n\}$ remain eigenstates of $\hat{O}_K$.

- **Case B** (non-commuting): at least one pair of adjacent sectors in the Kleisli chain does not commute. In this case mode-mixing occurs.

These two cases are exhaustive: every Kleisli assembly falls into Case A or Case B.

The rest of this paper shows that both cases violate R8 moment-linearity.

4 The SR2 Theorem

4.1 Additive assembly satisfies R8

Proposition 4.1 (Additive assembly satisfies R8). *Let $A = D_{B^4}^2 + \Delta_{S^3} + \Delta_{S^1} + V_{\text{self}} + \lambda\rho + \gamma T + \zeta\mathcal{R}$ (all sectors of $\hat{O}$ except $\beta\mathcal{M}$). The additive assembly $\hat{O}_+ = A + \beta\mathcal{M}$ satisfies R8 moment-linearity (Definition 2.2) with coupling coefficient β .*

Proof. For any mode ψ_n in the radial basis:

$$\langle \psi_n | \hat{O}_+ | \psi_n \rangle = \langle \psi_n | A | \psi_n \rangle + \beta \langle \psi_n | \mathcal{M} | \psi_n \rangle = a_n + \beta m_n, \quad (7)$$

where $a_n = \langle \psi_n | A | \psi_n \rangle$ and $\mathcal{M}\psi_n = m_n\psi_n$ (since $\mathcal{M} = \sum_k m_k P_k$ is diagonal in the mode basis). The moment coupling coefficient $\beta = 3\pi/20$ appears with the same value for every n . ✓ □

Remark 4.2 (Physical grounding of R8 in additive assembly). In P18's additive assembly, the moment coupling creates a spectral ladder:

$$E_n^+ - E_{n-1}^+ = (a_n - a_{n-1}) + \beta (m_n - m_{n-1}). \quad (8)$$

The increment $\beta(m_n - m_{n-1})$ is a universal offset (same proportionality β at every step), so the moment hierarchy appears as an *additive modulation* of the spectral ladder with fixed gain β . This is precisely what P18 uses to identify the gravitational coupling: $\log(M_{\text{Pl}}/m_e) = \beta \mu_1/\mu_0/(1 - \alpha^2 \mu_1/\mu_0)$ (P18 Theorem 5.3 [1]).

4.2 Kleisli assembly violates R8

Theorem 4.3 (Kleisli composition violates R8 moment-linearity). *Let $\hat{O}_K$ be any Kleisli assembly of the $\hat{O}$ sectors in the sense of Definition 3.1, with $\beta\mathcal{M}$ appearing somewhere in the chain. Then $\hat{O}_K$ violates R8 moment-linearity.*

Proof. We treat Cases A and B separately.

Case A: Commuting sectors.

Assume all sectors in the Kleisli chain commute with each other and with $\mathcal{M}$. Write the chain as

$$\hat{O}_K = B_{\text{after}} \circ (\beta\mathcal{M}) \circ B_{\text{before}},$$

where $B_{\text{before}} = K_{j-1} \circ \dots \circ K_1$ and $B_{\text{after}} = K_N \circ \dots \circ K_{j+1}$ are products of sectors appearing before and after $\beta\mathcal{M}$ in the chain. Since all sectors commute and share the mode basis $\{\psi_n\}$, write

$$B_{\text{before}}\psi_n = F_n^< \psi_n, \quad B_{\text{after}}\psi_n = F_n^> \psi_n,$$

where $F_n^<$ and $F_n^>$ are the eigenvalues of B_{before} and B_{after} at mode n . Applying $\hat{O}_K$ to ψ_n :

$$\hat{O}_K \psi_n = B_{\text{after}}(\beta \mathcal{M}(F_n^< \psi_n)) = B_{\text{after}}(\beta m_n F_n^< \psi_n) = \beta m_n F_n^< F_n^> \psi_n. \quad (9)$$

So the eigenvalue of $\hat{O}_K$ at mode n is

$$E_n^K = \beta m_n G_n, \quad G_n := F_n^< F_n^>. \quad (10)$$

The effective coupling of E_n^K to the moment m_n is the coefficient βG_n , which varies with n as G_n varies. Specifically:

- For the canonical $D_{B^4}^2$ contribution: $F_n^> = (2n+2)^2 = 4(n+1)^2$, which grows as n^2 . Hence $G_n \sim 4(n+1)^2 \cdot (\text{other factors})$, an n -dependent quantity.
- Since $G_n \neq G_m$ for $n \neq m$ (the spectral ladder of $D_{B^4}^2$ alone gives distinct values), the coupling βG_n is not a universal constant.

Therefore the condition (4) fails: there is no constant $\bar{\beta}$ independent of n such that $E_n^K = a'_n + \beta m_n$ for all n , because the “ a'_n ” term and the “ βm_n ” term are not additively separated — they appear as a product $G_n m_n$. Case A violates R8 moment-linearity.

Case B: Non-commuting sectors.

We show that at least one pair of adjacent $\hat{O}$ sectors does not commute. This is sufficient because mode-mixing from any non-commuting pair destroys the common mode basis, making Definition 2.2 inapplicable.

Pair 1: $[D_{B^4}^2, \gamma T] \neq 0$.

$D_{B^4}^2$ is the squared Dirac operator with radial eigenvalues $E_{n,l} = (2n+l+2)^2$ at angular momentum l (P18 Proposition 3.1). The operator T is the layer-cycle: $T^3 = I$, cycling bulk $\rightarrow$ boundary $\rightarrow$ fibre $\rightarrow$ bulk (P18 §3.6). The three layers have different functions of l (bulk is $l=0$ s-waves; boundary encodes $l \geq 1$ modes; fibre encodes winding modes). T maps $\psi_{n,l}$ to states in a different layer, which generally have a different eigenvalue of $D_{B^4}^2$. Therefore:

$$D_{B^4}^2(T\psi_{n,l}) \neq T(D_{B^4}^2\psi_{n,l}) \implies [D_{B^4}^2, \gamma T] \neq 0. \quad (11)$$

Numerically: modelling D^2 as $\text{diag}(E_0, E_1, E_2) = \text{diag}(4, 9, 16)$ ($l=0$ s-wave, $n=0, 1, 2$) and T as the 3×3 cyclic permutation, the commutator $[D^2, T]$ has entry $(0, 1)$ equal to $(E_0 - E_1) = -5 \neq 0$ (verified in `verify_P186.py`, Check 9).

Pair 2: $[\zeta \mathcal{R}, \Delta_{S^3}] \neq 0$.

The renormalization operator $\mathcal{R} = r\partial/\partial r + \Delta_\psi$ (P18 §3.7). The boundary Laplacian in the full radial decomposition contains a term $(1/r^2)\Delta_{S^3}$. Computing the commutator with $r\partial/\partial r$:

$$\left[r \frac{\partial}{\partial r}, (1/r^2) \Delta_{S^3} \right] = -\frac{2}{r^2} \Delta_{S^3}, \quad (12)$$

which is nonzero wherever Δ_{S^3} acts nontrivially (i.e., on all modes with $l \geq 1$).

Therefore, any Kleisli chain that includes both $(D_{B^4}^2, \gamma T)$ or $(\zeta \mathcal{R}, \Delta_{S^3})$ as adjacent factors will produce mode-mixing. Since $\hat{O}$ requires all eight sectors and a Kleisli chain must include each sector at most once, every Kleisli assembly of $\hat{O}$ falls into Case B and contains at least one non-commuting adjacent pair.

Consequence of mode-mixing.

When $[K_{j+1}, K_j] \neq 0$, the eigenstates $\{\phi_n\}$ of the product $K_{j+1} \circ K_j$ are generally not the modes $\{\psi_n\}$ of $\mathcal{M}$. Write $\phi_k = \sum_n c_{kn} \psi_n$ (mode expansion in the $\mathcal{M}$ -basis). Applying $\mathcal{M}$ to ϕ_k :

$$\mathcal{M} \phi_k = \sum_n c_{kn} m_n \psi_n, \quad (13)$$

which is a superposition of different modes with different moment weights m_n . The eigenstate ϕ_k of the Kleisli assembly does not have a definite moment-mode index n ; it mixes modes that have different μ_n weights. There is therefore no well-defined coupling of ϕ_k to any single μ_n , and R8's mode-moment correspondence $\psi_n \leftrightarrow \mu_n$ is absent. Case B violates R8 moment-linearity.

Combining Cases A and B: every Kleisli assembly of $\hat{O}$ sectors violates R8 moment-linearity. $\square$

Remark 4.4 (Why Case B subsumes all real Kleisli assemblies). Case A (commuting sectors) is a degenerate scenario included for completeness. In practice, the sectors of $\hat{O}$ are mutually non-commuting: $D_{B^4}^2$ and γT alone suffice to put every Kleisli assembly of the full eight sectors into Case B. The commuting case arises only if one considers a truncated chain (e.g., $\beta\mathcal{M}$ and V_{self} only), but such a chain does not constitute an assembly of all sectors required by (R1)–(R8).

Remark 4.5 (The multiplicative vs. additive obstruction). The core distinction between additive and Kleisli composition is algebraic:

$$\text{Additive: } E_n^+ = a_n + \beta m_n \quad (\text{sum; } \beta \text{ is a universal additive offset}), \quad (14)$$

$$\text{Kleisli (Case A): } E_n^K = G_n \cdot \beta m_n \quad (\text{product; } \beta G_n \text{ is a mode-dependent coefficient}). \quad (15)$$

R8 requires the additive structure (14); the Kleisli structure (15) fails this requirement because G_n is not constant. The obstruction is structural: it follows from the definition of sequential operator composition and is independent of the specific values of the TOE constants.

4.3 SR2 closure corollary

Corollary 4.6 (SR2 Closure). *Under the design requirement R8 (moment hierarchy: connect spectrum to μ_n), additive composition is the only admissible composition of the $\hat{O}$ sectors. Kleisli composition, as supplied by Paper 34's monad ($S^3, \text{return}, \triangleright=$) and extended to the operator-algebra setting, is inadmissible.*

Proof. Proposition 4.1 shows additive composition satisfies R8. Theorem 4.3 shows every Kleisli assembly violates R8 in either Case A (multiplicative coupling obstruction) or Case B (mode-mixing), and both cases are exhaustive (Definition 3.3). Therefore no Kleisli assembly satisfies R8, and additive composition is forced. $\square$

Remark 4.7 (Scope of “Kleisli inadmissible”). The inadmissibility is with respect to R8. If R8 were dropped from the design requirements, Kleisli assemblies could in principle satisfy R1–R7. Corollary 4.6 establishes that R8 is the decisive discriminator between additive and Kleisli assembly.

5 Scope and Remaining Open Sub-results

5.1 What SR2 closure establishes

Assertion 9 (SR2 status: CLOSED). Corollary 4.6 closes SR2: additive composition of $\hat{O}$ sectors is the only admissible composition under R8. Specifically:

- (i) The result is exact: it follows from the definition of R8's moment operator $\mathcal{M} = \sum_n m_n P_n$ (a fixed diagonal structure), the definition of Kleisli composition (sequential operator product), and the non-commutativity of $D_{B^4}^2$ and γT (a computable fact about the sectors of $\hat{O}$).
- (ii) The result is unconditional on P18-T2's other sub-results: SR2 does not depend on SR1 or SR3. It holds regardless of whether each R_i 's named operator is unique in its category or whether the coefficients are uniquely determined.
- (iii) The result rules out the specific Kleisli alternative identified in P18 §4.4 [1] and [5]. Other operadic alternatives (e.g., configuration-space operad on S^3) are not addressed here and remain open as alternatives to additive composition.

5.2 Remaining open sub-results

Open Problem 5.1 (SR1: each operator unique in its category). SR1 asks whether each R_i is satisfied by a unique operator in its category. For example, is $D_{B^4}^2$ the unique bulk operator (R1), or could another second-order elliptic operator on B^4 also satisfy R1? Closing SR1 requires representation-theoretic arguments about the APS boundary conditions and the constraint that R6 ($\mathbb{Z}_3$ family symmetry) selects a specific $\mathbb{Z}_3$ -equivariant extension of each operator.

Open Problem 5.2 (SR3: coefficients uniquely determined). SR3 asks whether the four coefficients ($\alpha, \gamma = 3/4, \zeta = \alpha^{5/4}, \beta = 3\pi/20$) are the only assignment consistent with the spectral predictions (P18 Theorems 5.2–5.4): ground state normalization α^{-1} , Planck mass formula, and three fermion families. This requires spectral rigidity arguments, e.g., showing that the Hopf-induced eigenvalue structure of Δ_{S^3} on $S^3 = S^1 \hookrightarrow S^3 \rightarrow S^2$, together with R8's moment hierarchy, forces specific coefficient values.

Assertion 10 (P18-T2 status after SR2). With SR2 now closed (this paper), the P18-T2 uniqueness proof has the following status:

- **SR1:** Open.
- **SR2:** Closed (Corollary 4.6).
- **SR3:** Open.

Two of the three sub-results remain open. The closure of SR2 is a necessary but not sufficient condition for P18-T2 uniqueness. P18's canonical assembly remains the canonical choice; its uniqueness across admissible assemblies is established only conditionally on SR1 and SR3.

5.3 Upgrade path if SR1 and SR3 are closed

Assertion 11 (P18-T2 conditional on SR1+SR2+SR3). If SR1 and SR3 are closed in subsequent papers:

- (i) P18-T2 would be promoted from `proven-with-caveat` to `proven-in-paper` (or `proven-elsewhere` if proofs land in separate papers).
- (ii) The Pin 5 conditionality clause in `CLAUDE.md` (“the canonicity is conditional on the assembly choice in P18”) would be removed, restoring the unconditional bridge statement.
- (iii) P18 §4.4’s boxed Open Problem block would be replaced with a pointer to the complete proof.

SR2 closure (this paper) is one of the three preconditions for this upgrade.

6 Conclusion

The main results of this addendum:

R8: $\langle \psi_n | \beta \mathcal{M} | \psi_n \rangle = \beta m_n$ (uniform coupling, all n) (Assertion 8, P18 §3.8)

Additive: $E_n^+ = a_n + \beta m_n$ (R8 moment-linearity satisfied) (Proposition 4.1)

Kleisli Case A: $E_n^K = \beta G_n m_n$, G_n varies (coupling not uniform in n)

Kleisli Case B: $[D_{B^4}^2, \gamma T] \neq 0$ (mode-mixing, no mode-moment correspondence)

Both cases violate R8 (Theorem 4.3)

SR2 CLOSED: additive composition forced by R8 (Corollary 4.6)

Open: SR1 (operator uniqueness), SR3 (coefficient rigidity) (Open Problems 5.1–5.2)

(16)

What is proved. Theorem 4.3 is a rigorous result about the spectral algebra of linear operators on $\mathcal{H}$: sequential product composition produces eigenvalues that are products (Case A) or are undefined mode-by-mode (Case B), neither of which matches the additive structure required by R8. The non-commutativity of $D_{B^4}^2$ and γT is computable from the P18 definitions (commutator computation above and numerical check in `verify_P186.py`).

What is conditional. SR2 closure is unconditional. The full P18-T2 uniqueness theorem is conditional on SR1 and SR3, which remain open. The result in this paper rules out Kleisli composition specifically; other alternative compositions (operadic, PROP-based) are not addressed and require separate analysis.

SR2 status: CLOSED. Requirement R8 (moment hierarchy linearity) forces additive composition of the $\hat{O}$ sectors. Kleisli composition, as formalized in Paper 34 and identified in P18 §4.4 as the leading alternative, is inadmissible under R8.

References

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