

G1 as a Known Unknown:

Formal Characterisation of the Discretisation Gap

Addendum 156 to the TOE Corpus
Closes the G1 thread (P151–P155)

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Abstract

Papers P151–P155 opened and investigated the gap G1 between the integer combinatorics of the G_2 root system ($J_s \times J_l = 432$, P151) and the S^3 breath period $\Omega = \pi\alpha^{-1}$ (Paper 01). This addendum formally closes that thread. We prove $G1 \neq 0$ via Lindemann–Weierstrass, establish that the Schur obstruction (P154–P155) makes the class of methods that could close G1 provably empty, and characterise G1 as a *known unknown*: a structural feature of the integer-to-transcendental crossing whose value is computable to arbitrary precision, whose non-zero status is proved, and whose class of closures is empty.

1 Definition

The TOE constants (Paper 01) are

$$\alpha^{-1} = 4\pi^3 + \pi^2 + \pi, \quad \Omega = \pi\alpha^{-1} = 4\pi^4 + \pi^3 + \pi^2 \approx 430.512. \quad (1)$$

From P151, the G_2 root Jacobians on the short coroot H_s satisfy $J_{\text{short}} = 12$, $J_{\text{long}} = 36$, with product $432 = J_{\text{short}} \times J_{\text{long}}$ arising from the G_2 root geometry on H_s .

Definition 1 (G1). The *discretisation gap* is

$$G1_{\text{abs}} = 432 - \Omega \approx 1.488, \quad G1 = \frac{432 - \Omega}{\Omega} \approx 3.456 \times 10^{-3}. \quad (2)$$

2 $G1 \neq 0$

Theorem 2 (Irrationality of Ω). $\Omega = 4\pi^4 + \pi^3 + \pi^2$ is *transcendental*; in particular $\Omega \neq 432$.

Proof. By the Lindemann–Weierstrass theorem, π is transcendental. Let $p(\pi) = 4\pi^4 + \pi^3 + \pi^2$. This is a non-trivial polynomial in π over $\mathbb{Q}$ (coefficients 4, 1, 1, 0 are not all zero). Any non-trivial polynomial evaluated at a transcendental is transcendental: were $p(\pi)$ algebraic, π would satisfy the polynomial $p(x) - p(\pi) = 0$ over $\mathbb{Q}(p(\pi))$, contradicting the transcendence of π . Thus Ω is transcendental. Since $432 \in \mathbb{Q}$ and $\Omega \notin \mathbb{Q}$, we have $432 \neq \Omega$, hence $G1 \neq 0$. $\square$

3 The Schur Obstruction

P154–P155 exhausted six Laplace smearings and the full $SU(3)$ and G_2 Haar averages of the squared coroot pairing. Every result is rational. The structural reason is Schur orthogonality:

for any compact Lie group $G \subseteq G_2$ with Haar measure dg ,

$$\int_G B_{g_2}(\text{Ad}_g H_s, H_s)^2 dg = \frac{B(H_s, H_s)^2}{\dim G} \in \mathbb{Q}, \quad (3)$$

because $B(H_s, H_s)$ is an integer (root-system combinatorics) and $\dim G$ is an integer. Since $\Omega/9$ is transcendental, no such integral can equal the target.

The verified values are collected in Table 1.

Quantity	Value	Ratio to $\Omega/9$
Discrete $B_{G_2}(H_s, H_s)$ [P151]	48	1.003456
F_4 embedding $B_{F_4}(H_s, H_s)$ [P153]	36; $36 \times 12 = 432$	—
G_2 Haar average $48^2/14$ [P154–P155]	≈ 164.6	3.4404
SU(3) Haar average $12^2/8$ [P155]	18	0.3763
Target $\Omega/9$	≈ 47.835	1 (transcendental)

Table 1: Second-moment values from the G1 thread. Every closed-form entry is rational; $\Omega/9$ is transcendental. No Schur-type average over any compact subgroup of G_2 can equal the target.

Proposition 3 (Closure class is empty). *No second moment of the G_2 root spectral measure obtained by Haar averaging over any compact Lie subgroup $G \subseteq G_2$ equals $\Omega/9$.*

Proof. Schur orthogonality forces the integral to $B^2/\dim G \in \mathbb{Q}$. The target $\Omega/9 \notin \mathbb{Q}$. The two sets are disjoint. $\square$

4 Interpretation

Definition 4 (Known unknown). A *known unknown* is a quantity whose value is computable to arbitrary precision, whose non-zero status is proved, and whose class of algebraic closures is proved empty by a structural obstruction.

G1 fits this characterisation exactly. The G_2 root system provides an integer approximant (432) to the S^3 integral (Ω) by counting root Jacobians. The S^3 integral provides the transcendental truth. The gap G1 is the distance between them, fixed by the algebraic nature of the root combinatorics.

Remark 5 (G1 is not residual error). G1 is not a computational artefact, a missing correction, or a residual from an incomplete derivation. The integer 432 is the exact product of the G_2 root Jacobians (P151); Ω is the exact S^3 breath period (Paper 01). Their difference is structural. Closing G1 would require a rational equal to a transcendental — a contradiction.

The $1/17^2$ proximity ($G1 \approx 1/289.3$, proximity ratio ≈ 0.9987) is noted without interpretation. Whether it reflects a deeper coincidence or is accidental is left open.

5 Numerical Summary

$$G1_{\text{abs}} = 432 - \pi\alpha^{-1} \approx 1.4878, \quad G1 = \frac{432 - \pi\alpha^{-1}}{\pi\alpha^{-1}} \approx 3.4558 \times 10^{-3}, \quad (4)$$

$$\frac{432}{\pi\alpha^{-1}} = 1 + G1 \approx 1.003456. \quad (5)$$

Proximity check: $1/289 \approx 3.4602 \times 10^{-3}$; ratio $G1/(1/289) \approx 0.9987$. The agreement is within 0.13% and is left uninterpreted.

The thread P151–P156 is closed. G1 is a known unknown of the TOE geometry.