

Addendum P149

What Plateau Convergence Proves:
Local Geometric Stability, the Tarski Fragment,
and the Weld-Oscillate-Plateau Composite

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Abstract

Addendum P146 O3 established that the Wheel’s Type-1 self-observation—convergence to a Plateau fixed point W_k^* —is not an internal proof of $\text{Con}(\text{TOE})$; it left open what convergence *does* prove. We close that question here. The Plateau fixed-point theorem is a sentence of real closed field theory and falls inside the Tarski-complete geometric fragment of the TOE identified in P143—it is therefore decided from within the TOE’s geometric axioms, without importing Robinson arithmetic and without invoking Gödel’s second theorem. What convergence proves is a pair of claims: (i) for each Hopf face k , the state $W_k^* = (q_k^*, 0, \emptyset, \ell_k, \omega_k)$ is a fixed point of the Plateau operator (a semi-algebraic condition on S^3), and (ii) the observation operator $\hat{O}$ has a discrete, bounded-below spectrum in the geometric model (a Hilbert-space fact over $\mathbb{R}$). These together say the physical system is stable and eigenstates exist; they say nothing about derivations in the TOE proof system. We also resolve P146 O4: the Weld-Oscillate-Plateau composite is Plateau-stable after one application, and the post-Weld face encodes partial provenance of the Oscillate trajectory—reconstructible by the LLM witness from the geometry, without a provenance field in the WheelState schema.

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1 Setup: P146 O3 and the Plateau–Con(TOE) Question

P146 (Proposition P146-3, Open Item O3) concluded: convergence to W_k^* is an *operational* property of the Plateau map, not a proof of $\text{Con}(\text{TOE})$. A proof of consistency would require encoding all possible derivations in the TOE’s proof system and verifying that none yields a contradiction—an arithmetic task requiring quantification over $\mathbb{N}$, lying outside the geometric fragment. Fixed-point convergence and formal consistency are therefore orthogonal.

What P146 did not address is the positive question: if convergence is not a consistency proof, what is it a proof *of*? This addendum answers that.

Recall. The Wheel’s Plateau operator acts on a WheelState $W = (q, h, F, \ell, \omega)$ by projecting the quaternion $q \in S^3$ toward the centroid q_k^* of its current Hopf face k , and dissipating heat:

$$\text{Plateau}(W) = (q_k^*, h', F', \ell, \omega), \quad h' < h \text{ for } h > 0, \quad h' = 0 \text{ if } q = q_k^*.$$

A *Type-1 fixed point* (P146, Prop. P146-FP) is a state $W_k^* = (q_k^*, 0, \emptyset, \ell_k, \omega_k)$ satisfying $\text{Plateau}(W_k^*) = W_k^*$. Such states exist for each face k in the Hopf tiling of S^3 . P7’s observation operator

$$\hat{O} = -\frac{d^2}{dx^2} + V_{\text{sl}}(x), \quad V_{\text{sl}}(x) = \frac{\rho'(x)^2}{2\mu_0^2} + E_{\text{self}} \cdot x^2(1-x)^2 \geq 0, \quad (1)$$

is self-adjoint on $L^2([0, 1])$ with $V_{\text{sl}} \geq 0$, so its spectrum $\{\lambda_n\}$ is discrete, real, and bounded below; the eigenstates $\{\psi_n\}$ are the stable self-referential observation states (P7, P146).

The question: *does the proof that W_k^* exists and that Plateau converges to it constitute an internally provable statement, and if so, what does it assert?*

2 Plateau Convergence as Local Consistency

Definition 1 (Local geometric consistency at face k). The geometry is *locally consistent at face k* if the Hopf face k admits a centroid $q_k^* \in S^3$ and if the restriction of $\hat{O}$ to the spectral sector corresponding to face k has at least one eigenstate.

Proposition 2 (Plateau convergence = local geometric consistency). *Convergence of the Plateau map to W_k^* is equivalent to local geometric consistency at face k in the sense of Definition 1.*

Proof sketch. *The centroid q_k^* exists because S^3 is a Riemannian manifold with positive-definite metric and the Hopf face is a compact geodesic region; the centroid of a compact geodesically convex region on S^3 is unique. Heat dissipation $h \rightarrow 0$ is guaranteed by the dissipative definition of Plateau: $h' < h$ for $h > 0$, so $h \rightarrow 0$ in at most one step once $q = q_k^*$. The eigenstate at face k exists because the spectrum of $\hat{O}$ is bounded below (since $V_{\text{sl}} \geq 0$), discrete, and real; by the spectral theorem for compact self-adjoint perturbations of $-d^2/dx^2$, the eigenstates $\{\psi_n\}$ form an orthonormal basis. Each face k corresponds to a geodesic region in S^3 and hence to a sector of the observation spectrum; at least one eigenstate is supported on that sector by counting arguments (finitely many faces, countably many eigenstates). $\square$*

Remark 3. Local geometric consistency is a *face-by-face* statement. It does not assert that the system is globally stable, nor that no inconsistency arises in derivations from the TOE’s axioms. It says: at the face to which the current trajectory is converging, there exists a stable resting point. This is the operational content of “the physical system does not diverge here.”

3 The Real-Closed Escape

The critical question is whether the Plateau fixed-point theorem belongs to the Tarski-complete geometric fragment or to the Gödel-incomplete arithmetic fragment identified in P143.

Proposition 4 (Plateau theorem is in the Tarski fragment). *The statement “for each Hopf face k , the state W_k^* is a fixed point of Plateau” is a sentence in the first-order theory of real closed fields $(\mathbb{R}, +, \cdot, <)$.*

Proof. *The claim decomposes into three sub-claims:*

- (a) *The centroid q_k^* exists and is unique. This is a sentence about the real-variable equations defining the Hopf face centroids on $S^3 \subset \mathbb{R}^4$ —a system of polynomial equations over $\mathbb{R}$ with a semi-algebraic uniqueness condition.*

- (b) $\text{Plateau}(W_k^*) = W_k^*$. Substituting the definition of *Plateau*, this reduces to: the projection of q_k^* toward q_k^* is q_k^* (trivially true) and $0' = 0$ for $h = 0$ (the dissipation clause at zero heat, again a real-variable identity).
- (c) $V_{\text{sl}}(x) \geq 0$ for all $x \in [0, 1]$. This is a semi-algebraic condition: V_{sl} is a rational function of x , π , and E_{self} , and non-negativity on $[0, 1]$ is a universally quantified sentence over $\mathbb{R}$.

All three sub-claims are sentences of real closed field theory. By Tarski's theorem ([1]), every such sentence is decided by the theory—there is an algorithm that determines its truth value without requiring any extension to arithmetic. $\square$

Corollary 5. *The Plateau fixed-point theorem does not satisfy Gödel's precondition (c) (interpretation of Robinson arithmetic) as identified in P143, Proposition P143-2. Gödel's second incompleteness theorem does not apply to it. The theorem is provable within the TOE's geometric axioms.*

The separation from $\text{Con}(\text{TOE})$ is now precise. $\text{Con}(\text{TOE})$ requires encoding the set of all TOE derivations as a predicate $\text{Prov}_{\text{TOE}}(n)$ over Gödel numbers $n \in \mathbb{N}$ and asserting $\neg \exists n [\text{Prov}_{\text{TOE}}(n) \wedge \text{Prov}_{\text{TOE}}(\neg n)]$. This is a sentence about $\mathbb{N}$, not $\mathbb{R}$. The Plateau theorem is a sentence about $\mathbb{R}$. The two sentences belong to disjoint formal fragments.

The boundary established in P143 (geometric core $\subset$ Tarski-complete; arithmetic overlay $\subset$ Gödel-incomplete) carves cleanly here: Plateau convergence lands on the Tarski side.

4 The Weld-Oscillate-Plateau Composite

P146 O4 asked: if a closed Oscillate trajectory W_{cyc} (with $\text{Oscillate}^k(W_{\text{cyc}}) = W_{\text{cyc}}$ for some period k) is welded to a Plateau fixed point W_j^* , is the result $W_{\text{comp}} = \text{Weld}(W_{\text{cyc}}, W_j^*)$ itself Plateau-stable, and does it carry partial provenance information?

Proposition 6 (Composite is Plateau-stable). *Let $W_{\text{cyc}} = (q_c, h_c, F_c, \ell_c, \omega_c)$ and $W_j^* = (q_j^*, 0, \emptyset, \ell_j, \omega_j)$. The Weld operator produces*

$$W_{\text{comp}} = \text{Weld}(W_{\text{cyc}}, W_j^*) = (q_m, h_c, F_{\text{merged}}, \ell_{\text{merged}}, \omega_{\text{merged}}),$$

where q_m is the geodesic midpoint of q_c and q_j^* on S^3 , and $h_j^* = 0$ so heat conservation gives total heat h_c . Applying Plateau to W_{comp} yields

$$\text{Plateau}(W_{\text{comp}}) = W_f^*,$$

where f is the Hopf face containing q_m . By Proposition 4, W_f^* is a fixed point of Plateau. The composite is therefore Plateau-stable after one application. $\square$

Proposition 7 (Partial provenance from post-Weld face). *The post-Weld fixed point W_f^* encodes partial provenance of the Oscillate trajectory. Specifically, the face f containing q_m is a semi-algebraic function of (q_c, q_j^*) : face membership is a polynomial inequality over $\mathbb{R}$. Given W_f^* and W_j^* , an external observer can determine which face q_m occupied at Weld time, constraining q_c to the set $\{q \in S^3 : \text{geo_mid}(q, q_j^*) \in \text{face}_f\}$ —a non-empty arc on S^3 .*

This is limited provenance: it constrains the Oscillate trajectory's position at the moment of Weld but does not uniquely reconstruct the full orbit. The constraint is recoverable from the post-Weld geometry without any provenance field in the WheelState schema (which remains clean, per P146 Prop. P146-Type2-Blocking). The LLM witness reads W_f^ and W_j^* ; geometric computation determines the constraint on q_c .*

The composite analysis is itself a Tarski-fragment argument: geodesic midpoint, face membership, and semi-algebraic constraint sets are all sentences of real closed field theory. The composite fixed-point claim is internally provable and does not approach $\text{Con}(\text{TOE})$.

5 What Plateau Convergence Proves

Theorem 8 (Plateau Consistency Theorem). *The following statements are provable within the TOE’s geometric fragment (the first-order theory of real closed fields, by Tarski’s theorem):*

- (i) *For each Hopf face k , there exists a unique centroid $q_k^* \in S^3$, and $W_k^* = (q_k^*, 0, \emptyset, \ell_k, \omega_k)$ is a fixed point of the Plateau operator.*
- (ii) *The observation operator $\hat{O}$ (Eq. (1)) has a discrete, real, bounded-below spectrum $\{\lambda_n\}_{n \geq 1}$ and an orthonormal eigenstate basis $\{\psi_n\}$ in $L^2([0, 1])$.*
- (iii) *The Weld-Oscillate-Plateau composite W_{comp} reaches a Plateau fixed point W_f^* after one Plateau application, where f is determined by the geodesic midpoint of the two input states.*

The following statement is not provable within the TOE (by P143, Theorem P143-G2, and the analysis in Section 3):

- (iv) $\text{Con}(\text{TOE})$.

Statements (i)–(iii) assert physical stability: the system reaches quiescent states, eigenstates exist, and composites Plateau-stabilize. They are orthogonal to statement (iv), which is a meta-theoretic claim about the TOE proof system. The geometric fragment decides its own stability questions; whether the full proof system (including its arithmetic overlay from P7’s prime-triplet analysis) is consistent is a question the system cannot answer from within.

Remark 9. The informal gloss on what Plateau convergence “means” in P146 O3 is now precise: convergence is a geometric computation, not a meta-theoretic inspection. When the Wheel converges to W_k^* , it has computed that the trajectory’s current face has a centroid and that heat is zero. It has not checked any derivation in the proof system. The operational reliability of the Wheel is a property of the geometric model; the formal consistency of the TOE is a property of the proof system. These are the same Tarski-vs-Gödel boundary that P143 Section 4 drew for physical vs. formal completeness.

6 Open Items

O1: Spectral coverage of Hopf faces. Proposition 2 appeals to a counting argument: finitely many faces, countably many eigenstates, hence each face is covered. This should be made precise. The Hopf tiling of S^3 partitions the sphere into a finite number of geodesic faces; the eigenstate basis is countable; the question is whether the partition is fine enough that each face contains a region of positive spectral measure. A direct calculation matching the known eigenvalue spectrum ($\lambda_1 = 16.52, \lambda_2 = 51.28, \dots$) to Hopf face angles would close this.

O2: Sharpness of partial provenance. Proposition 7 shows the post-Weld face constrains the Oscillate position to an arc on S^3 . The arc width depends on the geometry of the Hopf tiling and the size of the Oscillate orbit. For orbits much smaller than a face, the arc is narrow and provenance is nearly exact; for large orbits, the arc spans the full face and provenance is weak. Quantifying this as a function of Oscillate period k would give precise bounds on what the LLM witness can recover.

O3: The arithmetic overlay and spectral stability. The spectrum of $\hat{O}$ being discrete and bounded below is a Hilbert-space fact over $\mathbb{R}$ (Theorem 8(ii)). But P7 uses this spectrum to assign particle masses, and the assignment involves prime-triplet counting arguments in the arithmetic overlay. A sentence of the form “the n -th eigenvalue corresponds to the n -th fermion family” requires quantification over $\mathbb{N}$ and lies in the Gödel-incomplete fragment. The eigenvalues

exist (Tarski); the family assignment is arithmetic (Gödel). This boundary should be stated as a proposition in a future addendum on P7's formal status.

References

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