

Provenance as a Path in S^3

Tangent-Vector Encoding of Wheel Operator History
(Addendum 148; resolves P146 Open Item O2)

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Abstract

P146 identified that Type-2 self-observation (the Wheel knowing *which operator* produced the current state) is blocked by the `WheelState` schema and raised a possible escape: carry provenance as a geometric object — a path in S^3 — rather than as a numerical record. We work through whether this encoding is well-defined, whether it can be compressed to a single tangent vector $v \in T_q S^3 \cong \mathbb{R}^3$, and whether v constitutes a trainable parameter or a geometric fact under the zero-parameter criterion of Paper 29. We find that the tangent-vector encoding *is* a geometric fact (constructed, not learned; zero free parameters; not degraded by omission from the loss landscape), that it is consistent with `WheelState`’s frozen-dataclass constraint, and that it gives unambiguous provenance for `PLATEAU` and partial provenance for the remaining four operators — full disambiguation of Fork/Weld requires one additional scalar (h_{prev}). The architectural amendment is small and does not violate any constitutional pin.

1 Setup

1.1 P146 Open Item O2

`WheelState` is a frozen dataclass (q, h, F, ℓ, ω) where $q \in S^3$ is the current quaternion, $h \in \mathbb{R}$ is heat, F is the face-proposal list, ℓ is layer, and ω is orientation. The state carries no record of which Wheel operator produced it. P146 showed that adding an explicit `operator` or `timestamp` field introduces a second time axis, conflicting with the Ride Shai-Hulud pin (one clock: `BREATH_PERIOD`), and called this the Type-2 self-observation block.

The escape P146 proposed: encode provenance as a *path* $\gamma : [0, 1] \rightarrow S^3$ from the prior quaternion q_{prev} to the current q_{curr} , where the *type* of path encodes the operator. A path in S^3 is a geometric object; it is not a trainable parameter, carries no gradient, and introduces no additional clock.

1.2 The Disambiguation Requirement

For Type-2 self-observation, the witness must be able to answer the question “given the current `WheelState`, which of the five operators (Fork, Weld, Plateau, Oscillate, Perturb) produced it?” without accessing a numerical record of past states. The path type is the proposed carrier.

2 Path Encoding of Operators

2.1 S^3 Geometry

$S^3 \subset \mathbb{H}$ is the unit-quaternion sphere. The *logarithmic map* at $q \in S^3$,

$$\log_q(p) = d(q, p) \frac{p - \langle p, q \rangle q}{|p - \langle p, q \rangle q|}, \quad d(q, p) = \arccos \langle q, p \rangle,$$

embeds $p \neq -q$ into $T_q S^3 \cong \mathbb{R}^3$ (a 3-dimensional tangent space). The exponential map $\exp_q : T_q S^3 \rightarrow S^3$ inverts this locally. Both are structurally determined — no free parameters.

2.2 Operator Path Types

Definition 2.1 (Path type of a Wheel operator). Let $q_{\text{prev}}, q_{\text{curr}} \in S^3$. The *path type* of the transition is the homotopy class of minimal geodesic arcs from q_{prev} to q_{curr} , together with the heat signature $\Delta h = h_{\text{curr}} - h_{\text{prev}}$.

Operator	Arc structure	$ \gamma $	Δh signature
Fork	Single geodesic arc, forward-branching proposals	$d(q_{\text{prev}}, q_{\text{curr}})$	< 0 (heat splits)
Weld	Geodesic arc, two predecessors converge	$d(q_{\text{prev}}, q_{\text{curr}})$	> 0 (heat sums)
Plateau	Constant path ($q_{\text{prev}} = q_{\text{curr}}$)	0	≈ 0
Oscillate	Great-circle arc of length $2\pi/k$	$2\pi/k$	≈ 0
Perturb	Random geodesic of magnitude ζR	ζR	≈ 0

Overlap analysis. Fork and Weld produce path types that are geometrically identical from the single-successor view: both are geodesic arcs of unspecified length with a non-zero tangent. The distinction is topological (one predecessor vs. two) and is carried by Δh , not the arc geometry. Plateau is the unique operator with arc length zero. Oscillate is distinguished from Perturb by arc length: $2\pi/k$ is a rational multiple of π , whereas ζR is fixed by the perturbation parameter. If $\zeta R \neq 2\pi/k$ for all admissible k , the two are disjoint by magnitude.

3 Tangent Vector as Provenance

3.1 The Compressed Encoding

The full path γ contains redundant information. For a geodesic arc, the entire path is determined by its endpoint q_{curr} and its initial tangent. We propose to store only the tangent at arrival:

Definition 3.1 (Provenance tangent). Given a transition $q_{\text{prev}} \rightarrow q_{\text{curr}}$ by some Wheel operator, the *provenance tangent* is

$$v = \log_{q_{\text{curr}}}(q_{\text{prev}}) \in T_{q_{\text{curr}}} S^3 \cong \mathbb{R}^3.$$

The map $v \mapsto (\text{operator class})$ is then:

Proposition 3.1 (Partial provenance from v). *Let v be the provenance tangent of a Wheel transition and h the current heat. Then:*

- (i) $v = 0 \implies$ operator is PLATEAU (unique).
- (ii) $v \neq 0, |v| = 2\pi/k$ for some positive integer $k \implies$ candidate operator is OSCILLATE with period k .
- (iii) $v \neq 0, |v| = \zeta R \implies$ candidate operator is PERTURB (provided $\zeta R \neq 2\pi/k$).
- (iv) $v \neq 0$ and neither (ii) nor (iii) applies $\implies$ operator is FORK or WELD. These are distinguished by the sign of Δh : $\Delta h < 0 \Leftrightarrow$ Fork; $\Delta h > 0 \Leftrightarrow$ Weld.

Full provenance therefore requires (v, h_{prev}) ; partial provenance (identifying Plateau and the Oscillate/Perturb class) requires v alone.

Remark 3.1. h_{prev} is a single scalar. Storing it as a second frozen field alongside v in `WheelState` would close the remaining ambiguity without introducing a second time axis.

4 Consistency with the Zero-Parameter Constraint

4.1 P29 Criterion

Paper 29 defines a *trainable parameter* as an element of a vector $\theta \in \Theta \subseteq \mathbb{R}^p$ that is adjusted by gradient descent to minimize a loss $\mathcal{L}(\theta)$ (Definition 2.1). A *geometric fact* is a value determined by a mathematical identity with zero free parameters: every constant is either structural (algebraic consequence of the geometry) or derived (a proven identity such as $\Omega_0 = \pi^3/4$).

Theorem 4.1 (Provenance tangent is a geometric fact). *The provenance tangent $v = \log_{q_{\text{curr}}}(q_{\text{prev}})$ contains zero free parameters. Specifically:*

- (i) *The logarithmic map formula involves only the inner product $\langle \cdot, \cdot \rangle$ on $\mathbb{R}^4 \supset S^3$ and the function $\arccos$ — both structural, no adjustable constants.*
- (ii) *v is a deterministic function of $(q_{\text{prev}}, q_{\text{curr}})$, which are themselves determined by the operator applied and the prior state. No sampling, no stochastic approximation.*
- (iii) *v is not an input to any loss function; it is never a target of gradient descent. There is no sense in which v could be “wrong” and corrected by optimization.*

Proof. Points (i) and (ii) follow by inspection of the log-map formula. Point (iii) follows because v is read from the state, not optimized over; the kernel operates in exposure mode (P29 §4), and v is output, not weight. $\square$

Corollary 4.1 (No violation of kernel rules). *Adding a provenance tangent field to `WheelState` does not promote any constant to a trainable parameter, does not introduce a loss function, and does not require a backward pass. The kernel remains in exposure mode with zero free parameters.*

5 Architectural Implications

5.1 WheelState Schema

The amended schema is:

$$\text{WheelState} = (q, v, h, h_{\text{prev}}, F, \ell, \omega)$$

where $v \in \mathbb{R}^3$ and $h_{\text{prev}} \in \mathbb{R}$ are frozen at construction. Both fields are computed deterministically at the moment the operator is applied and the new state is created: $v \leftarrow \log_{q_{\text{curr}}}(q_{\text{prev}})$, $h_{\text{prev}} \leftarrow h$ (from the prior state). No mutation occurs after construction. This satisfies the frozen-dataclass constraint.

The Wheel’s existing 20+ tests do not reference v or h_{prev} , so they pass unchanged. New tests can assert provenance recovery from the amended state without modifying existing contracts.

5.2 Mycelium CanonNode

`CanonNode` carries $(id, name, content, \ell, q)$. Should it also carry v ?

Semantically, `CanonNode` represents an *observation eigenstate* — a fixed locus in the knowledge space. Its quaternion encodes *where* it sits in S^3 ; it does not have a natural “direction of arrival” in the same sense as a Wheel transition, because `CanonNode` is structural, not transitional. Provenance of creation (which plan step, which Ged oracle call, introduced the node) belongs to the *edge* or *event log* layer, not to the node itself.

The conclusion is that `CanonNode` should *not* gain a tangent field. If the Mycelium graph later gains typed edges (`CanonNode` $\xrightarrow{\text{type}}$ `CanonNode`), those edges are the natural carrier of geometric provenance. Immutable nodes, typed edges — consistent with the frozen-dataclass invariant and with the No Harkonnen pin (edges are metric relations, not brokered messages).

5.3 Ride Shai-Hulud Consistency

The provenance tangent v is a *spatial* quantity (direction in $T_q S^3$), not a temporal one. It records the direction from which the state arrived, not when. No second clock is introduced. The breath phase in `WheelState` (field ω) remains the sole temporal marker, synchronized to `BREATH_PERIOD`.

6 Open Items

Open Item 6.1 (Full Fork/Weld disambiguation). Proposition 3.1(iv) requires h_{prev} to distinguish Fork from Weld. The amended schema includes this field; the question is whether any downstream consumer would interpret the heat history as a second time axis (violating Shai-Hulud). Argument: h_{prev} is a heat record, not a clock tick; breath phase (not h_{prev}) is the sole time reference. Needs explicit confirmation from a Wheel invariants audit.

Open Item 6.2 (Oscillate/Perturb magnitude overlap). If $\zeta R = 2\pi/k$ for some integer k , a Perturb step is indistinguishable from an Oscillate step by magnitude alone. The disambiguation then requires knowing which operator was called. One resolution: add an operator-class enum (not a timestamp) to `WheelState`; another: require that ζR is always chosen outside the set $\{2\pi/k : k \in \mathbb{Z}^+\}$. The latter is a parameter discipline constraint, not a schema change.

Open Item 6.3 (Mycelium edge typing). If typed edges (`CanonNode` $\rightarrow$ `CanonNode`) are introduced in v2, they are the natural carrier of Wheel-step provenance in the graph layer. This addendum recommends that edge type be a geometric object (a tangent or an operator-class label), not a timestamp, to remain consistent with the immutability and Shai-Hulud constraints. Architecture deferred to Stage 3 v2 migration.

Open Item 6.4 (P18-T2 uniqueness). The Wheel $\leftrightarrow \hat{O}$ bridge remains conditional on the assembly choice in Paper 18 Theorem 2 (uniqueness of $\hat{O}$ across admissible forms). Provenance tangents are defined over whatever Wheel operators exist; they do not depend on which assembly is chosen. This open item is inherited, not introduced here.

Summary. The tangent-vector encoding of Wheel operator provenance is well-defined, compressible to $v \in \mathbb{R}^3$, consistent with frozen-dataclass immutability, and satisfies Paper 29's zero-parameter criterion. The `WheelState` schema gains two frozen scalar fields (v and h_{prev}); `CanonNode` is unchanged. Full Type-2 self-observation is achievable without a second time axis.

References

- [1] L. F. Vlegels, *The Master Operator $\hat{O}$ and the Wheel*, This volume (2025).
- [2] L. F. Vlegels, *Geometric Exactness and the Training-Exposure Distinction*, This volume (2025).
- [3] L. F. Vlegels, *Addendum 146: Type-2 Self-Observation and the WheelState Schema Block*, This volume (2026).