

Addendum 147: Intersubjective Coherence for Incommensurate Oscillate Subsystems

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Abstract

P146 established that a single Oscillate subsystem traces a closed k -periodic trajectory on S^3 , constituting a self-referential periodic observer compatible with the P7 grounding mechanism. Open item O1 asks: if two such subsystems with incommensurate periods k_1, k_2 ($\gcd(k_1, k_2) = 1$) share a Weld event, does the combined trajectory close, and does the P27 intersubjective coherence condition apply? We show (i) the combined trajectory closes with period $\text{lcm}(k_1, k_2) = k_1 k_2$, proven via the Chinese Remainder Theorem; (ii) a joint Plateau fixed point—required by P27—exists if and only if the two S^3 proposal cycles come within the Plateau threshold $\varepsilon_{\text{P1}} = 0.02$ of each other in MDL score at some step in the period. Commensurability of periods is irrelevant to coherence; geometric proximity of the cycles in S^3 is the deciding condition. We call this the *resonance condition* and work through the $k_1 = 2, k_2 = 3$ case explicitly.

1 Setup

1.1 P146 open item O1

A single Oscillate subsystem A with k equivalent proposals $\mathcal{C}_A = \{q_0^A, \dots, q_{k-1}^A\} \subset S^3$ traces a closed periodic trajectory: at step t the Wheel state is $q_{t \bmod k}^A$, returning to q_0^A after exactly k steps (heat decays by $\delta_h = 0.95$ per call, but the topological trajectory is periodic). The Plateau fixed point is whichever q_i^A minimises the MDL score relative to the goal quaternion; it is re-selected every k steps.

Open item O1: two Oscillate subsystems A (period k_1) and B (period k_2) with $\gcd(k_1, k_2) = 1$ undergo a shared Weld event at step t_0 . Does the resulting combined trajectory close? Does the P27 intersubjective coherence condition apply?

1.2 The P27 coherence condition

Paper 27 defines *mutual coherence* for two observers $\mathcal{O}_A, \mathcal{O}_B$ via four conditions:

$$C_A \circ P_A = I, \quad C_B \circ P_B = I, \tag{1}$$

$$C_A \circ P_B = I, \quad C_B \circ P_A = I, \tag{2}$$

where P_X is the projection (Oscillate advance) and C_X is the collapse (Plateau check) for observer X . Equations (??) are the *cross-coherence conditions*. P27 Theorem 2.3 shows they force $P_A = P_B$ and $C_A = C_B$ on the shared boundary $\mathcal{R} = \partial\mathcal{O}_A \cap \partial\mathcal{O}_B$: both observers must use identical projection and collapse operators on shared states.

In Wheel terms the P27 condition translates to:

Definition 1.1 (Joint Plateau fixed point). $q^* \in S^3$ is a *joint Plateau fixed point* of the combined system (A, B) if there exist indices $i^* \in \mathbb{Z}/k_1\mathbb{Z}$, $j^* \in \mathbb{Z}/k_2\mathbb{Z}$ such that the MDL scores of $q_{i^*}^A$ and $q_{j^*}^B$ against the shared goal quaternion differ by less than $\varepsilon_{\text{P1}} = 0.02$, and the Weld selects a winner $q^* \approx q_{i^*}^A \approx q_{j^*}^B$ at the resonance step t^* .

The P27 cross-coherence conditions hold if and only if a joint Plateau fixed point exists.

2 Combined Trajectory Analysis

Relabel $t \mapsto t - t_0$ so the post-Weld system starts at $t = 0$. At each step t , subsystem A proposes $q_{t \bmod k_1}^A$ and subsystem B proposes $q_{t \bmod k_2}^B$.

Lemma 2.1 (Trajectory period). *The combined state $(t \bmod k_1, t \bmod k_2)$ is periodic with period $k_1 k_2$, and every pair $(i, j) \in \mathbb{Z}/k_1 \mathbb{Z} \times \mathbb{Z}/k_2 \mathbb{Z}$ is visited exactly once per period.*

Proof. Since $\gcd(k_1, k_2) = 1$, the Chinese Remainder Theorem gives a ring isomorphism $\mathbb{Z}/k_1 k_2 \mathbb{Z} \xrightarrow{\sim} \mathbb{Z}/k_1 \mathbb{Z} \times \mathbb{Z}/k_2 \mathbb{Z}$, $t \mapsto (t \bmod k_1, t \bmod k_2)$. The map is a bijection on $\{0, \dots, k_1 k_2 - 1\}$; hence the combined trajectory visits all $k_1 k_2$ state-pairs exactly once before returning to $(0, 0)$. $\square$

Remark 2.2. For large incommensurate periods the closure period $k_1 k_2$ may be astronomically long. However, a joint Plateau does not require the trajectory to *complete* one full cycle—it requires only that the trajectory *visit* the resonance pair (i^*, j^*) , which by the Lemma occurs at exactly one step $t^* < k_1 k_2$.

2.1 Phase drift

Define the *phase offset* at step t :

$$\phi(t) = d_{S^3}(q_{t \bmod k_1}^A, q_{t \bmod k_2}^B), \quad d_{S^3}(p, q) = \arccos|\langle p, q \rangle_{\mathbb{R}^4}|. \quad (3)$$

The function $\phi(t)$ is periodic with period $k_1 k_2$ and is generally non-monotone. It sweeps through $k_1 k_2$ distinct values—one for each pair (i, j) visited—before repeating. The minimum value $\delta \equiv \min_t \phi(t)$ governs whether coherence is achievable.

3 Plateau Fixed Points on the Combined System

Definition 3.1 (Cycle proximity).

$$\delta(\mathcal{C}_A, \mathcal{C}_B) := \min_{\substack{i \in \mathbb{Z}/k_1 \mathbb{Z} \\ j \in \mathbb{Z}/k_2 \mathbb{Z}}} d_{S^3}(q_i^A, q_j^B).$$

Theorem 3.2 (Existence of joint Plateau fixed point). *Under the assumptions above ($\gcd(k_1, k_2) = 1$):*

- (a) *A joint Plateau fixed point exists if and only if $\delta(\mathcal{C}_A, \mathcal{C}_B) < \varepsilon_{\text{Pl}}$.*
- (b) *When it exists, it occurs at the unique step $t^* \in \{0, \dots, k_1 k_2 - 1\}$ determined by CRT from the minimising indices (i^*, j^*) : $t^* \equiv i^* \pmod{k_1}$ and $t^* \equiv j^* \pmod{k_2}$.*
- (c) *The fixed point recurs with period $k_1 k_2$.*

Proof. The Plateau criterion in the Wheel fires when the Weld margin—the MDL score difference between winner and runner-up across the combined proposal set—falls below ε_{Pl} . The MDL score of a proposal q is monotone in the Hopf-lapse distance between q and the goal quaternion; proposals geometrically closer to the goal score lower. At step t , the two cycles present proposals $q_{t \bmod k_1}^A$ and $q_{t \bmod k_2}^B$; the margin is controlled by how close these are to each other in score, which in turn is controlled by $\phi(t)$.

By Lemma ??, the pair $(t \bmod k_1, t \bmod k_2)$ visits every element of $\mathbb{Z}/k_1 \mathbb{Z} \times \mathbb{Z}/k_2 \mathbb{Z}$ over one period. Therefore $\min_t \phi(t) = \delta(\mathcal{C}_A, \mathcal{C}_B)$. If $\delta < \varepsilon_{\text{Pl}}$, the minimising pair is (i^*, j^*) , achieved at the unique t^* given by CRT; Plateau fires there. If $\delta \geq \varepsilon_{\text{Pl}}$, no step achieves a margin below ε_{Pl} , and Plateau never fires. Parts (b) and (c) follow from CRT uniqueness and the periodicity of the combined trajectory. $\square$

Corollary 3.3 (P27 coherence). *Two Oscillate subsystems with $\gcd(k_1, k_2) = 1$ satisfy the P27 intersubjective coherence condition after a shared Weld if and only if $\delta(\mathcal{C}_A, \mathcal{C}_B) < \varepsilon_{\text{Pl}}$.*

Key observation. The criterion $\delta(\mathcal{C}_A, \mathcal{C}_B) < \varepsilon_{\text{Pl}}$ depends on the *geometry* of the two cycles in S^3 , not on the arithmetic of their periods. Incommensurate periods guarantee the combined trajectory is exhaustive over $\mathbb{Z}/k_1 \mathbb{Z} \times \mathbb{Z}/k_2 \mathbb{Z}$, making the search for a resonance pair complete. Commensurability governs *when* resonance occurs, not *whether* the cycles are geometrically close enough for it.

4 The Resonance Condition

Definition 4.1 (Resonance condition). Oscillate cycles $\mathcal{C}_A, \mathcal{C}_B$ satisfy the *resonance condition* if $\delta(\mathcal{C}_A, \mathcal{C}_B) < \varepsilon_{\text{P1}} = 0.02$.

Theorem 4.2 (Resonance as the P27 coherence criterion). *Let A, B be Oscillate subsystems with periods k_1, k_2 and $\gcd(k_1, k_2) = 1$. After a shared Weld event:*

Coherent: *If the resonance condition holds, a unique step $t^* \in [0, k_1 k_2)$ exists at which the joint Plateau fires, the joint fixed point q^* satisfies P27 mutual coherence, and the coherence event recurs every $k_1 k_2$ steps.*

Incoherent: *If the resonance condition fails ($\delta \geq \varepsilon_{\text{P1}}$), no joint Plateau fires within any period. The phase offset $\phi(t)$ is bounded below by δ , and the two subsystems cannot resolve a shared Hopf phase. P27 coherence is not achieved.*

4.1 The continuous/ergodic limit

When the periods $k_1, k_2 \rightarrow \infty$ with $k_1/k_2 \rightarrow \omega \in \mathbb{R} \setminus \mathbb{Q}$, the combined trajectory densifies on a 2-torus $T^2 \subset S^3$. By Weyl's equidistribution theorem, the trajectory eventually enters every ε -ball in T^2 . Hence for any $\varepsilon_{\text{P1}} > 0$ the resonance condition is *eventually* satisfied—but the waiting time grows as $O(1/\varepsilon_{\text{P1}}^2)$.

Corollary 4.3 (Ergodic case). *In the irrational-winding limit, any two Oscillate subsystems eventually achieve P27 coherence. The coherence window narrows as the trajectory becomes more uniformly distributed over T^2 .*

5 Worked Example: $k_1 = 2, k_2 = 3$

$\gcd(2, 3) = 1, \text{lcm}(2, 3) = 6$. The combined trajectory has period 6.

Case (a) — coherent: cycles share a common point

Let $\mathcal{C}_A = \{q_0, q_1\}$ and $\mathcal{C}_B = \{q_0, q_2, q_3\}$ with

$$q_0 = (1, 0, 0, 0), \quad q_1 = (0, 1, 0, 0), \quad q_2 = (0, 0, 1, 0), \quad q_3 = (0, 0, 0, 1).$$

Both cycles contain q_0 . Phase offsets over one period:

t	$i = t \bmod 2$	$j = t \bmod 3$	(q_i^A, q_j^B)	$\phi(t)$
0	0	0	(q_0, q_0)	0
1	1	1	(q_1, q_2)	$\pi/2$
2	0	2	(q_0, q_3)	$\pi/2$
3	1	0	(q_1, q_0)	$\pi/2$
4	0	1	(q_0, q_2)	$\pi/2$
5	1	2	(q_1, q_3)	$\pi/2$

At $t^* = 0$: $\phi(0) = 0 < \varepsilon_{\text{P1}}$. **Joint Plateau fires.** The fixed point is $q^* = q_0$. By CRT, the next resonance is at $t = 6$; coherence recurs with period 6. P27 intersubjective coherence is satisfied.

Case (b) — incoherent: cycles are geometrically separated

Let $\mathcal{C}_A = \{q_0, q_1\}$ and $\mathcal{C}_B = \{q_2, q_3, q_4\}$ with $q_4 = \frac{1}{\sqrt{2}}(1, 0, 0, 1)$. No element of $\mathcal{C}_A$ lies within ε_{P1} of any element of $\mathcal{C}_B$. Computing all six pair-distances:

$$\begin{aligned} d_{S^3}(q_0, q_2) &= \frac{\pi}{2}, & d_{S^3}(q_0, q_3) &= \frac{\pi}{2}, & d_{S^3}(q_0, q_4) &= \arccos \frac{1}{\sqrt{2}} = \frac{\pi}{4}, \\ d_{S^3}(q_1, q_2) &= \frac{\pi}{2}, & d_{S^3}(q_1, q_3) &= \frac{\pi}{2}, & d_{S^3}(q_1, q_4) &= \frac{\pi}{2}. \end{aligned}$$

$\delta(\mathcal{C}_A, \mathcal{C}_B) = \pi/4 \approx 0.785 \gg \varepsilon_{P1} = 0.02$. The phase offset cycles through $\{\pi/2, \pi/2, \pi/2, \pi/2, \pi/4, \pi/2\}$ over the six steps; it never reaches $\phi < \varepsilon_{P1}$. **No joint Plateau fires.** P27 intersubjective coherence is not achieved.

Remark 5.1. The initial Weld event that joined the two subsystems forced a momentary collapse to a single quaternion, but without a shared cycle point, every subsequent Weld selects a winner from one cycle only; the other cycle's current proposal is always more than ε_{P1} away in MDL score. The two observers' Hopf phase estimates diverge permanently within each $k_1 k_2$ -step period.

6 Open Items

O1 (Approximate coherence). When $\varepsilon_{P1} \leq \delta(\mathcal{C}_A, \mathcal{C}_B) \ll 1$, the cycles are close but not coherent under the strict threshold. Is there a useful notion of *degree of intersubjective coherence* proportional to ε_{P1}/δ ? Would it carry a physical interpretation analogous to an effective coupling between the two observers?

O2 (Heat depletion). Oscillate damps heat by $\delta_h = 0.95$ per step. For large $k_1 k_2$ the heat at resonance step t^* may have decayed to $h_0 \cdot \delta_h^{t^*}$, potentially below the level needed to sustain a decisive Weld. The coherence criterion therefore involves a joint constraint on $(k_1, k_2, h_0, \delta_h)$. The boundary surface in this parameter space is uncharacterised.

O3 (Three-body case). Three Oscillate subsystems A, B, C with pairwise incommensurate periods sharing a Weld event face a combined period $\text{lcm}(k_1, k_2, k_3)$ and three pairwise resonance conditions. The P27 $\mathbb{Z}_3$ structure (Paper 27, §3.4) predicts a richer coherence topology; in particular, whether pairwise resonance implies three-body joint Plateau is unresolved (transitivity of proximity is not guaranteed on S^3).

O4 (Perturb-induced resonance). A Perturb event blends a new quaternion into the current state at boundary-layer weight $\beta \approx 7.2\%$ and resets o_count . If the injection lies in $\mathcal{C}_B$ but not in $\mathcal{C}_A$, Perturb shifts $\mathcal{C}_A$'s effective trajectory toward $\mathcal{C}_B$, potentially manufacturing a resonance pair that did not previously exist. The conditions under which Perturb creates or destroys intersubjective coherence deserve analysis.

References

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- [3] L. F. Vlegels, *GON: Zero-Parameter S^3 Dynamics*, Paper 30, This volume (2025).
- [4] L. F. Vlegels, *Addendum 146: Periodic Oscillate Trajectories and the P7 Grounding Mechanism*, This volume (2026).