

Addendum P145

The Scope Theorem: What the TOE Predicts and Why

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Abstract

Addendum P141 established that 85.1% of the PDG hadronic vacuum polarisation $\Delta\alpha_{\text{had}}^{\text{PDG}}$ is decoupled from the $J_3(\mathbb{O})$ eigenvalue spectrum by construction. This raises the structural question: what, precisely, is the TOE a theory of? We derive a *Scope Theorem*: the TOE is the theory of the *algebraic skeleton* of gauge physics, comprising exactly those observables whose hadronic corrections are expressible as linear functionals of the discrete pole spectrum $\mathcal{S}_{\text{had}}^{J_3(\mathbb{O})}$. The boundary criterion is the $\mathbf{P}_{\text{EW}}$ -projectability of the hadronic dispersion kernel: an observable is TOE-algebraic if and only if its kernel $K_O(s)$ is effectively supported at scales $s \gtrsim M_\rho^2$ where the $J_3(\mathbb{O})$ poles dominate. Algebraic observables are mutually correlated (all functions of the same $J_3(\mathbb{O})$ structure); dynamical ones are independent inputs. The non-derivability of QCD dynamics from $J_3(\mathbb{O})$ is itself a theorem, not an observation: it follows from a dimension argument on $L^2(\mathbb{R}_+)$. Applied to the muon $g-2$: any tension between theory and experiment is structurally confined to the dynamical sector and cannot originate in the algebraic EW precision observables.

Contents

1 Setup

Addenda P121–P141 completed the TOE electroweak precision programme. The quantities α^{-1} , M_W , M_Z , G_F , $\sin^2 \theta_W$, $\alpha_s(M_Z)$, the five resonance masses $\{M_\rho, M_\omega, M_\phi, M_{J/\psi}, M_\Upsilon\}$, and the CKM Wolfenstein parameters were all derived from the $J_3(\mathbb{O})$ discrete spectrum and the master operator $\hat{O}$ (P18, P20) to per-mille accuracy or better. In every case the prediction rests on the same structure: eigenvalues and Peirce-block residues of $J_3(\mathbb{O})$, with no QCD Fock-space input.

Simultaneously, P136–P141 showed that the multi-pion continuum, the charm–bottom hadronic running, and the hadronic light-by-light contributions are *structurally outside* this framework by Peirce block orthogonality (P141, Theorem 1). The TOE accounts for 14.9% of $\Delta\alpha_{\text{had}}^{\text{PDG}}$; the remaining 85.1% is annihilated by $\mathbf{P}_{\text{EW}}$.

The pattern raises the deepest structural question: is this a limitation of the TOE, or a precise characterisation of what the TOE claims to be? The answer is the latter. The present addendum derives it formally and states it as a theorem.

2 Observable Classification

Let O be an observable whose hadronic correction takes the dispersive form

$$\Delta O_{\text{had}} = \int_{4m_\pi^2}^{\infty} K_O(s) \frac{\text{Im } \Pi_{\text{had}}(s + i0^+)}{\pi s^n} ds, \quad (1)$$

for a non-negative kernel K_O and integer $n \geq 1$.

Definition 1 (Observable classification). An observable O admitting the representation (??) is:

- *TOE-algebraic* if ΔO_{had} is well-approximated by replacing $\text{Im } \Pi_{\text{had}}$ with $\text{Im } \Pi_{\text{had}}^{\text{TOE}}$, the $\mathbf{P}_{\text{EW}}$ -projected correlator of P141; error is $O(\alpha_s/\pi)$.
- *TOE-dynamical* if the dominant support of $K_O(s) \text{Im } \Pi_{\text{had}}(s)/s^n$ lies in the branch-cut sector annihilated by $\mathbf{P}_{\text{EW}}$ (Lemma 1, P141).
- *Hybrid* if both sectors contribute at the $\gtrsim 10\%$ level.

Table ?? gives canonical examples.

Table 1: Classification of observables by TOE-accessibility. Q_{eff} is the effective scale at which $K_O(s)$ is peaked.

Observable	Physical content	Q_{eff}	Class
$\alpha^{-1} = 4\pi^3 + \pi^2 + \pi$	$\int_0^1 \rho dx$ (P3)	—	Algebraic
M_W, M_Z	EW Peirce eigenvalues (P49, P93)	M_Z	Algebraic
G_F	Fermi self-consistency (P123)	M_W	Algebraic
$\sin^2 \theta_W$	$SU(2)/U(1)$ dimension ratio (P42)	M_Z	Algebraic
$\alpha_s(M_Z)$	G_2 -sector matching (P86, P90)	M_Z	Algebraic
M_ρ, M_ω, M_ϕ	$J_3(\mathbb{O})$ resonance poles (P136)	M_V	Algebraic
CKM angles	Wolfenstein from $\mathbb{Z}_3$ (P82)	—	Algebraic
$a_\mu^{\text{HVP,LO}}$	Hadronic dispersion at μ -scale	m_μ	Dynamical
a_μ^{HLbL}	Four-photon loop at m_μ	m_μ	Dynamical
$ F_\pi(s) ^2, s < M_\rho^2$	Sub- ρ pion form factor (P141)	$< M_\rho$	Dynamical
$\alpha_s(\mu), \mu < M_Z$	QCD running below EW scale	μ	Dynamical
$\mu_{\text{conf}} = 313 \text{ MeV}$	Confinement onset (P120)	μ_{conf}	Hybrid

The confinement scale μ_{conf} is hybrid: its *value* is algebraically determined as the scale at which $\alpha_s = O(1)$ by G_2 -matching (P120), but the running of α_s between μ_{conf} and M_Z is a G_2 -sector dynamical process not yet derived from $J_3(\mathbb{O})$ alone.

3 The Boundary Criterion

Definition 2 ($\mathbf{P}_{\text{EW}}$ -projectability). An observable O is $\mathbf{P}_{\text{EW}}$ -projectable if

$$\int K_O(s) \frac{\text{Im } \Pi_{\text{had}}^{\text{SM}}(s)}{\pi s^n} ds \approx \int K_O(s) \frac{\text{Im } \Pi_{\text{had}}^{\text{TOE}}(s)}{\pi s^n} ds, \quad (2)$$

up to $O(\alpha_s/\pi)$ corrections. Since $\text{Im } \Pi_{\text{had}}^{\text{TOE}}(s) = \pi \sum_{V \in \mathcal{S}_{\text{had}}^{J_3(\mathbb{O})}} C_V \delta(s - M_V^2)$ (P141, eq. (3)), the right side reduces to

$$\Delta O_{\text{had}}^{\text{TOE}} = \sum_{V \in \mathcal{S}_{\text{had}}^{J_3(\mathbb{O})}} \frac{K_O(M_V^2) C_V}{M_V^{2n}}. \quad (3)$$

This formula is valid when K_O is slowly varying on the scale $\Gamma_V \ll M_V$.

Proposition 3 (Scale criterion). *An observable O is TOE-algebraic if and only if $K_O(s)$ has effective peak scale $Q_{\text{eff}}^2 \gtrsim M_\rho^2$. The natural boundary is $Q_* \approx M_\rho \approx 770 \text{ MeV}$, set by the lightest algebraic pole in $\mathcal{S}_{\text{had}}^{J_3(\mathbb{O})}$.*

Proof. For $Q_{\text{eff}}^2 \gtrsim M_\rho^2$, the integrand in (??) has dominant support at $s \gtrsim M_\rho^2$ where $\text{Im } \Pi_{\text{had}}^{\text{SM}} \approx \text{Im } \Pi_{\text{had}}^{\text{TOE}}$ by Theorem 1(a) of P141 (no branch cuts in $\Pi_{\text{had}}^{\text{TOE}}$). For $Q_{\text{eff}}^2 \ll M_\rho^2$, dominant support lies in the branch-cut sector $s \in [4m_\pi^2, M_\rho^2]$, which is orthogonal to $\mathbf{P}_{\text{EW}}$ by Lemma 1 of P141. $\square$

Illustration. The Fermi hadronic running uses kernel $K_\alpha(s) = (1 - s/M_Z^2)^{-1}$, peaked at $s \sim M_Z^2 \gg M_\rho^2$: algebraic (P135). The muon HVP uses $K_\mu(s) \approx m_\mu^2/(3s)$, peaked at $s \sim m_\mu^2$, a factor ~ 53 below M_ρ^2 : dynamical.

4 Scope Theorem

Theorem 4 (Scope Theorem). *Let $\mathcal{A}$ be the set of TOE-algebraic observables.*

- (a) **Algebraic completeness.** *Every $O \in \mathcal{A}$ is determined by the ten real numbers $\{M_V, C_V : V \in \mathcal{S}_{\text{had}}^{J_3(\mathbb{O})}\}$ together with the $J_3(\mathbb{O})$ group-theoretic data via equation (??) and the master operator $\hat{O}$.*
- (b) **Dynamical independence.** *Let $\mathcal{D}$ be the dynamical complement. The hadronic spectral function $\rho_{\text{had}}(s) \in L^2(\mathbb{R}_+)$ has infinite-dimensional function space; the algebraic data $\{M_V, C_V\}$ occupy a 10-dimensional parameter space. No map $\mathbb{R}^{10} \rightarrow L^2(\mathbb{R}_+)$ is surjective onto any open set; hence generic QCD dynamical observables are not determined by $J_3(\mathbb{O})$ alone.*
- (c) **Algebraic correlations.** *Any two $O_1, O_2 \in \mathcal{A}$ satisfy an algebraic relation $R(O_1, O_2, \{M_V\}, \{C_V\}) = 0$ derivable from $J_3(\mathbb{O})$. No such universal relation exists between any element of $\mathcal{A}$ and any element of $\mathcal{D}$.*
- (d) **Non-derivability of dynamics is a theorem.** *The non-derivability in (b) is not an empirical observation about the TOE's current state of development; it is a consequence of the dimension argument applied to the finite-rank map $\mathbb{R}^{10} \hookrightarrow L^2(\mathbb{R}_+)$. Completing the TOE corpus cannot change this: any extension that adds new algebraic content adds finitely many parameters, while $L^2(\mathbb{R}_+)$ remains infinite-dimensional.*

Proof sketch. (a) follows from Theorem 1 of P141 together with P82–P135. (b) and (d): $\rho_{\text{had}}(s)$ is an arbitrary non-negative function subject only to QCD unitarity; its function space is infinite-dimensional. A finite-rank map $\mathbb{R}^{10} \rightarrow L^2(\mathbb{R}_+)$ has image of Lebesgue measure zero; it cannot represent generic hadronic spectral functions. (c): All $O \in \mathcal{A}$ are rational functions of $\{M_V^2, C_V\}$ via (??); eliminating the pole parameters gives an algebraic relation. Elements of $\mathcal{D}$ depend on the $\mathbf{P}_{\text{EW}}$ -orthogonal branch-cut sector (P141, Lemma 1), which is independent of $\{M_V, C_V\}$. $\square$

Corollary 5 (Falsifiability structure). *All algebraic observables lie on a 10-dimensional algebraic submanifold of the EW precision data space. A single measurement placing two algebraic observables outside this submanifold falsifies the TOE. Measurements of dynamical observables, however, cannot falsify the algebraic sector: they probe independent degrees of freedom.*

The Scope Theorem resolves the apparent tension between the TOE's precision in the EW sector and its silence on hadronic dynamics. The silence is not a gap to be filled by future addenda; it is a structural boundary with a mathematical proof.

5 Muon $g-2$ as Test Case

The leading hadronic contribution to the muon anomalous magnetic moment is

$$a_\mu^{\text{HVP,LO}} = \frac{\alpha}{\pi} \int_{4m_\pi^2}^{\infty} K_\mu(s) \frac{\text{Im } \Pi_{\text{had}}(s)}{\pi s} ds, \quad (4)$$

with $K_\mu(s)$ peaked at $s \sim m_\mu^2 = (105.7 \text{ MeV})^2$. Since $m_\mu^2/M_\rho^2 \approx 1/53$, Proposition ?? classifies $a_\mu^{\text{HVP,LO}}$ as *TOE-dynamical*. The ρ -pole contributes $\approx 28\%$ of the total via vector meson dominance; the remaining $\approx 72\%$ arises from the sub-threshold pion form factor, which is outside $\mathcal{S}_3^{J_3(\mathbb{O})}$. The hadronic light-by-light piece a_μ^{HLbL} is also entirely dynamical: the relevant four-point loop momentum is $\sim m_\mu$, sub-confinement.

What the TOE predicts about the tension. By Theorem ??(c), algebraic and dynamical observables are independent. If experiment finds $a_\mu^{\text{exp}} \neq a_\mu^{\text{SM}}$, the discrepancy *cannot* originate in the algebraic sector: α , M_W , M_Z , and $\sin^2 \theta_W$ are independently verified to lie on the $J_3(\mathbb{O})$ algebraic submanifold, and these are the inputs to the leptonic and electroweak corrections in a_μ^{SM} . The discrepancy must therefore be located in either (i) the dynamical hadronic sector (HVP or HLbL) or (ii) a BSM contribution. The TOE provides no constraint between these two options; both are in $\mathcal{D}$.

Dispersive vs. lattice. The ongoing disagreement between lattice QCD evaluations of $a_\mu^{\text{HVP,LO}}$ and dispersive evaluations from e^+e^- cross-section data is a dispute *within* $\mathcal{D}$: both compute the same dynamical observable from different methods. Their disagreement is a QCD systematics question, not an algebraic-sector question. The TOE neither adjudicates nor contributes to its resolution.

6 Open Items

O1: G_2 renormalisation group. The confinement scale $\mu_{\text{conf}} = 313 \text{ MeV}$ is algebraically fixed (P120), but $\alpha_s(\mu)$ for $\mu_{\text{conf}} \leq \mu < M_Z$ requires the G_2 -sector β -function, not yet derived from $J_3(\mathbb{O})$ first principles. Completing the G_2 RG would promote $\alpha_s(\mu)$ from hybrid to fully algebraic, closing the largest hybrid observable in Table ??.

O2: Pion form factor from $\mathcal{P}_{12}$. Deriving $|F_\pi(s)|^2$ via Omnès unitarisation from the $\mathcal{P}_{12}$ Peirce sector (P113) would extend the algebraic boundary below the ρ peak and reduce the dynamical fraction of $a_\mu^{\text{HVP,LO}}$ from 72% toward a calculable residual. This would be the first closure of a currently dynamical observable.

O3: Explicit algebraic relations. Corollary ?? predicts algebraic relations among $\{M_W, M_Z, \sin^2 \theta_W, \alpha^{-1}, G_F\}$ beyond the SM tree-level identities. The TOE corrections are parametrised by the Peirce residues C_V ; deriving these relations explicitly would provide a new class of falsifiability tests distinct from individual-observable precision.

O4: Gravity in the scope picture. G (Newton) is algebraically determined (P17); the bulk B^4 carries dynamical gravitational degrees of freedom. The Scope Theorem structure should extend to gravity, with boundary S^3 observables algebraic and bulk dynamical observables in a gravitational $\mathcal{D}$. Making this precise would complete the scope picture across all four forces.

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