

Addendum P138

Pion Final-State Interactions and ρ - ω Interference:
Two Sub-GeV Corrections to $\Delta\alpha_{\text{had}}$ and the W -Boson Mass

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Abstract

Addendum P137 computed the hadronic vacuum polarisation from $J_3(\mathbb{O})$ spectral masses using the Gounaris-Sakurai (GS) running-width Breit-Wigner, finding $\Delta\alpha_{\text{had}}^{\text{lm GS}} = 3.731 \times 10^{-3}$ —a 9.2% undershoot of the Fermi target 4.110×10^{-3} . P137 identified two open sub-GeV mechanisms: pion final-state interactions (FSI) below $\sqrt{s} \approx 0.6$ GeV, and ρ - ω interference near the 1 GeV boundary. This addendum evaluates both corrections from first principles and determines whether they close the gap.

The FSI correction replaces the simple Breit-Wigner pion form factor $|F_\pi|_{\text{BW}}^2$ in the sub-threshold region $4m_\pi^2 \leq s \leq 0.36 \text{ GeV}^2$ with the Omnès-enhanced form factor, using the linear approximation $\delta_1^1(s) \approx (s/M_\rho^2) \cdot (\pi/2)$ for the P -wave $\pi\pi$ phase shift. This gives $\Delta\alpha_{\text{had}}^{\text{FSI}} = +2.182 \times 10^{-4}$, closing 57.6% of the gap.

The ρ - ω interference cross-term uses the TOE mixing amplitude $\varepsilon = -\alpha_s(M_Z)/3 = -0.03953$ from P118. With the correct cross-term formula $\Delta R_{\text{mix}}(s) = \frac{1}{4}\beta^3(s) |F_\pi|_{\text{BW}}^2(s) \cdot 2\varepsilon \cdot \text{Re}[B_\omega(s)]$, the dispersive integral gives $\Delta\alpha_{\text{had}}^{\rho\omega} = -2.316 \times 10^{-4}$. The negative sign arises because $\varepsilon < 0$ and the integration range extends further below M_ω^2 (where $\text{Re}[B_\omega] > 0$) than above it, so destructive interference dominates.

The two corrections nearly cancel: $\Delta\alpha_{\text{had}}^{\text{FSI}} + \Delta\alpha_{\text{had}}^{\rho\omega} = -1.3 \times 10^{-5}$. The combined P138 total is $\Delta\alpha_{\text{had}}^{\text{lm P138}} = 3.718 \times 10^{-3}$, leaving a residual of -9.55% against the Fermi target—marginally worse than P137's -9.2%. The W -boson mass is $M_W^{\text{P138}} = 80.383 \text{ GeV}$ (+0.008% vs PDG 80.377 GeV), indistinguishable from the P137 result. The gap between the GS estimate and the Fermi target cannot be closed by these two mechanisms; their near-cancellation is itself a structural result.

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1 Setup: the P137 Open Items

The dispersive formula for the hadronic contribution to the running of the fine-structure constant is

$$\Delta\alpha_{\text{had}}(M_Z^2) = \frac{\alpha}{3\pi} \int_{4m_\pi^2}^{\infty} \frac{R(s)}{s} \frac{M_Z^2}{M_Z^2 - s} ds, \quad (1)$$

where $R(s) = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma_{\mu\mu}$ and $\sigma_{\mu\mu} = 4\pi\alpha^2/(3s)$. Addendum P137 evaluated the $\rho(770)$ contribution using the Gounaris-Sakurai lineshape and found $\Delta\alpha_{\text{had}}^{\rho \text{ GS}} = 2.943 \times 10^{-3}$, giving a light-meson total $\Delta\alpha_{\text{had}}^{\text{lm GS}} = 3.731 \times 10^{-3}$. The Fermi target (P135) is 4.110×10^{-3} ; the gap is

$$\delta\Delta\alpha_{\text{had}} = 4.110 \times 10^{-3} - 3.731 \times 10^{-3} = 3.79 \times 10^{-4} \quad (9.22\% \text{ of target}). \quad (2)$$

P137 identified two structural reasons the GS integral may underestimate the true hadronic contribution in the sub-GeV domain. First, the GS propagator for the ρ models the pion form factor as a simple Breit-Wigner, which fails below $\sqrt{s} \approx 0.6 \text{ GeV}$ where pion-pion final-state interactions produce Omnès-factor corrections. Second, the ρ and ω are treated as independent resonances in P137, but isospin breaking (characterised in P118) generates an interference cross-term in the $\pi^+\pi^-$ channel that is absent from the separated NWA treatment of the ω . This addendum derives and evaluates both corrections.

2 Mechanism 1: Pion Final-State Interactions (Omnès Correction)

2.1 The Omnès enhancement below the ρ peak

Below the $\rho(770)$ peak, the pion electromagnetic form factor $F_\pi(s)$ receives a significant enhancement from rescattering of the final-state pion pair. The Omnès formula expresses this as

$$|F_\pi(s)|_{\text{Om}}^2 = |F_\pi(s)|_{\text{BW}}^2 \cdot \exp\left[\frac{2\delta_1^1(s)}{\pi} \ln \frac{\Lambda^2}{s}\right], \quad (3)$$

where $\delta_1^1(s)$ is the P -wave $\pi\pi$ scattering phase shift, $\Lambda \sim 1 \text{ GeV}$ is the Omnès cutoff, and $|F_\pi|_{\text{BW}}^2 = M_\rho^4/[(s - M_\rho^2)^2 + M_\rho^2\Gamma_\rho^2]$ is the BW form factor squared. In the sub-threshold region a linear approximation for the phase shift is accurate to a few percent: $\delta_1^1(s) \approx (s/M_\rho^2) \cdot (\pi/2)$, giving $2\delta_1^1/\pi = s/M_\rho^2$. With $\Lambda = 1 \text{ GeV}$ the Omnès factor simplifies to

$$\frac{|F_\pi|_{\text{Om}}^2}{|F_\pi|_{\text{BW}}^2} = \exp\left[\frac{s}{M_\rho^2} \ln \frac{1}{s}\right] = s^{-s/M_\rho^2}, \quad (4)$$

which grows from 1 at threshold ($s \rightarrow 0$) to ≈ 1.84 at $\sqrt{s} = 0.6 \text{ GeV}$.

2.2 FSI contribution to $\Delta\alpha_{\text{had}}$

The correction to the $e^+e^- \rightarrow \pi^+\pi^-$ cross-section ratio from FSI is

$$\Delta R_{\text{FSI}}(s) = \frac{1}{4} \beta^3(s) |F_\pi|_{\text{BW}}^2(s) [s^{-s/M_\rho^2} - 1], \quad (5)$$

where $\beta(s) = \sqrt{1 - 4m_\pi^2/s}$ is the pion velocity and the factor $1/4$ is the point cross-section normalisation for the charged-pion channel. The FSI contribution to $\Delta\alpha_{\text{had}}$ is then

$$\Delta\alpha_{\text{had}}^{\text{FSI}} = \frac{\alpha}{3\pi} \int_{4m_\pi^2}^{0.36 \text{ GeV}^2} \frac{\Delta R_{\text{FSI}}(s)}{s} \frac{M_Z^2}{M_Z^2 - s} ds. \quad (6)$$

The upper limit $\sqrt{s_{\text{hi}}} = 0.6 \text{ GeV}$ is the boundary below which the simple BW fails; above this energy the GS running-width propagator already incorporates the dominant lineshape effects. The integral is evaluated by 400-point Gauss-Legendre quadrature on five sub-intervals matched to the rapid growth of $|F_\pi|_{\text{BW}}^2$ near the ρ peak. Table 1 shows the sub-interval contributions.

Table 1: Sub-interval contributions to $\Delta\alpha_{\text{had}}^{\text{FSI}}$ from equation (6). All values in units of 10^{-4} . The dominant contribution comes from the highest sub-interval $[0.520, 0.600] \text{ GeV}$, where $|F_\pi|_{\text{BW}}^2$ grows rapidly toward the ρ peak.

$\sqrt{s}$ range (GeV)	$\Delta\alpha_{\text{sub}}^{\text{FSI}} (\times 10^{-4})$	Fraction
[0.279, 0.346]	0.052	2.4%
[0.346, 0.400]	0.151	6.9%
[0.400, 0.458]	0.304	13.9%
[0.458, 0.520]	0.520	23.9%
[0.520, 0.600]	1.155	52.9%
Total	2.182	100%

The FSI integral gives $\Delta\alpha_{\text{had}}^{\text{FSI}} = 2.182 \times 10^{-4}$, closing 57.6% of the P137 gap. The physical origin of the enhancement is the pion-pion rescattering below the ρ peak: the phase shift $\delta_1^1 > 0$ corresponds to attractive $\pi\pi$ interaction in the P -wave, which constructively amplifies the form factor. The correction grows from negligible near threshold to a factor of 1.84 in $|F_\pi|^2$ at $\sqrt{s} = 0.6 \text{ GeV}$; the corresponding correction to $R_{\pi\pi}$ is modulated by $\beta^3(s)$ and the dispersive kernel, concentrating most of the integral in the upper half of the integration domain.

3 Mechanism 2: ρ - ω Interference Cross-Term

3.1 Mixing amplitude from P118

Addendum P118 derived the ρ - ω mixing amplitude from the J_1 Peirce sector of $J_3(\mathbb{O})$:

$$\varepsilon = -\frac{\alpha_s(M_Z)}{C_\rho/C_\omega} = -\frac{0.1186}{3} = -0.03953, \quad (7)$$

where $\alpha_s(M_Z) = 0.1186$ (P106) is the strong coupling at the M_Z anchor and $C_\rho/C_\omega = 3$ is the electromagnetic charge ratio from the G_2 Cartan structure (P112). The negative sign was derived in P118 from the G_2 charge structure: the ρ^0 couples to the photon through the isovector combination $Q_u - Q_d = +1$, while the mixing enters the ω with the opposite sign, reflecting the opposite isospin of the two channels.

3.2 Cross-term in the pion form factor

Including ρ - ω mixing, the pion electromagnetic form factor takes the form

$$F_\pi(s) = B_\rho(s)[1 + \varepsilon B_\omega(s)], \quad B_V(s) = \frac{M_V^2}{s - M_V^2 - iM_V\Gamma_V}, \quad (8)$$

where $B_V(0) = -1$ by construction so that $F_\pi(0)$ remains normalised to unity at the leading-order level. The modification to $|F_\pi|^2$ relative to the pure ρ form factor is

$$\Delta|F_\pi|^2 = |B_\rho(s)|^2 |1 + \varepsilon B_\omega(s)|^2 - |B_\rho(s)|^2 \approx |B_\rho(s)|^2 \cdot 2\varepsilon \text{Re}[B_\omega(s)], \quad (9)$$

where the $\mathcal{O}(\varepsilon^2)$ term has been dropped. The real part of the ω propagator is

$$\text{Re}[B_\omega(s)] = \frac{M_\omega^2(M_\omega^2 - s)}{(M_\omega^2 - s)^2 + (M_\omega\Gamma_\omega)^2}. \quad (10)$$

This function is odd about $s = M_\omega^2$: it is positive for $s < M_\omega^2$ (below the ω pole) and negative for $s > M_\omega^2$. Its amplitude peaks at $s = M_\omega^2 \pm M_\omega\Gamma_\omega$ with magnitude $M_\omega/(2\Gamma_\omega) \approx 46$, reflecting the narrow width of the ω .

3.3 Dispersive integral for the cross-term

The cross-term contribution to $R(s)$ for the $\pi^+\pi^-$ channel is

$$\Delta R_{\text{mix}}(s) = \frac{1}{4} \beta^3(s) |F_\pi|_{\text{BW}}^2(s) \cdot 2\varepsilon \text{Re}[B_\omega(s)], \quad (11)$$

and the corresponding dispersive integral is

$$\Delta\alpha_{\text{had}}^{\rho\omega} = \frac{\alpha}{3\pi} \int_{4m_\pi^2}^{1\text{ GeV}^2} \frac{\Delta R_{\text{mix}}(s)}{s} \frac{M_Z^2}{M_Z^2 - s} ds. \quad (12)$$

The integral is evaluated by 600-point Gauss-Legendre quadrature on a fine grid of sub-intervals straddling the ω pole to resolve the narrow structure of $\text{Re}[B_\omega(s)]$.

Sign and magnitude. Since $\varepsilon < 0$ and $\text{Re}[B_\omega(s)] > 0$ below M_ω^2 , the integrand $\Delta R_{\text{mix}}(s)$ is negative for $s < M_\omega^2$ and positive for $s > M_\omega^2$. The integration range is asymmetric about M_ω^2 : the sub- ω interval spans $M_\omega^2 - 4m_\pi^2 \approx 0.535\text{ GeV}^2$, while the super- ω interval spans $1 - M_\omega^2 \approx 0.387\text{ GeV}^2$. The larger range below M_ω^2 causes the negative contribution to dominate. Numerically:

$$\Delta\alpha_{\text{had}}^{\rho\omega}|_{s \leq M_\omega^2} = -1.101 \times 10^{-3}, \quad \Delta\alpha_{\text{had}}^{\rho\omega}|_{s \geq M_\omega^2} = +8.69 \times 10^{-4},$$

giving a net

$$\Delta\alpha_{\text{had}}^{\rho\omega} = -2.316 \times 10^{-4}. \quad (13)$$

The negative sign has a clear physical interpretation: the $\varepsilon < 0$ mixing means the ω amplitude enters the pion form factor with a sign opposite to F_ρ . Below M_ω^2 , this is destructive interference, suppressing $|F_\pi|^2$. Since the ρ peak lies at $M_\rho^2 = 0.601\text{ GeV}^2 < M_\omega^2 = 0.613\text{ GeV}^2$, most of the spectral weight of the ρ contribution sits in the destructive region, and the net integral is negative. This is consistent with the narrow dip-peak structure observed in $\sigma(e^+e^- \rightarrow \pi^+\pi^-)$ near $\sqrt{s} \approx 782\text{ MeV}$ in BaBar and KLOE data: the cross section dips slightly below the ω pole (from destructive interference) and peaks slightly above it (constructive), with the integrated dip area exceeding the peak area because the ρ tail weights the sub- ω region more heavily.

4 Combined Result

4.1 Near-cancellation of the two mechanisms

The two corrections are almost equal in magnitude and opposite in sign:

$$\Delta\alpha_{\text{had}}^{\text{FSI}} = +2.182 \times 10^{-4} \quad (+57.6\% \text{ of gap}), \quad (14)$$

$$\Delta\alpha_{\text{had}}^{\rho\omega} = -2.316 \times 10^{-4} \quad (-61.1\% \text{ of gap}). \quad (15)$$

Their sum is

$$\Delta\alpha_{\text{had}}^{\text{FSI}} + \Delta\alpha_{\text{had}}^{\rho\omega} = -1.34 \times 10^{-5}, \quad (16)$$

a net correction of -3.5% of the gap—in the wrong direction. Adding this to the P137 GS baseline:

$$\Delta\alpha_{\text{had}}^{\text{lm P138}} = 3.731 \times 10^{-3} + (-1.34 \times 10^{-5}) = 3.718 \times 10^{-3}. \quad (17)$$

The residual gap against the Fermi target is

$$\Delta\alpha_{\text{had}}^{\text{lm P138}} - 4.110 \times 10^{-3} = -3.924 \times 10^{-4} \quad (-9.55\%), \quad (18)$$

marginally larger than P137's -9.2% gap. Table 2 collects the full comparison.

Table 2: Corrections to the P137 GS baseline and their effect on the Fermi gap. The FSI and $\rho\omega$ contributions are computed in this addendum; all other inputs are from P136–P137.

Contribution	$\Delta\alpha_{\text{had}} (\times 10^{-3})$	Gap fraction
P137 GS baseline	3.731	—
$\Delta\alpha_{\text{had}}^{\omega}$ (NWA, P136)	[0.281]	—
$\Delta\alpha_{\text{had}}^{\phi}$ (NWA, P136)	[0.507]	—
FSI Omnès correction (this work)	+0.218	+57.6%
ρ – ω cross-term (this work)	−0.232	−61.1%
Net P138 correction	−0.013	−3.5%
P138 total	3.718	−9.55% gap
Fermi target (P135)	4.110	—

4.2 Why the mechanisms cancel

The near-cancellation is not accidental. Both mechanisms operate on the same spectral region: the pion form factor below the ρ peak. The FSI (Omnès) enhancement increases $|F_{\pi}|^2$ in this region by the factor s^{-s/M_{ρ}^2} , which grows monotonically from threshold to the cutoff at $\sqrt{s} = 0.6 \text{ GeV}$. The ρ – ω cross-term decreases $|F_{\pi}|^2$ in the same region by $2\varepsilon \text{Re}[B_{\omega}(s)]$, which is also a smooth, monotonically growing function from threshold to M_{ω}^2 (since $\text{Re}[B_{\omega}]$ increases as $s \rightarrow M_{\omega}^2$ from below, and $\varepsilon < 0$). Both corrections are of order a few percent of $|F_{\pi}|^2$ in this domain and their competing effects nearly cancel in the dispersive integral.

The underlying reason for near-equality in magnitude is the parametric relationship between the two corrections. The Omnès enhancement is governed by $\delta_1^1 \sim s/M_{\rho}^2$, which at $s \approx M_{\omega}^2$ gives $\delta_1^1(M_{\omega}^2) \approx \pi/2 \cdot (M_{\omega}/M_{\rho})^2 \approx 0.51 \text{ rad}$, so that the Omnès factor at the ω mass is $\approx e^{0.51/\pi \cdot \ln(1/M_{\omega}^2)} \approx 1.87$. The ρ – ω correction at that same energy is $2|\varepsilon| \cdot M_{\omega}/(2\Gamma_{\omega}) \approx 2 \times 0.0395 \times 46 \approx 3.6$ — but this is the peak correction integrated over the narrow ω width, whereas the dispersive integral samples $\text{Re}[B_{\omega}]$ over the full range including the long Cauchy tails, which average out differently from the Omnès correction. The fact that these two unrelated physical effects—one from hadronic rescattering, the other from QCD isospin breaking—produce nearly identical magnitudes over the same spectral region suggests that the GS integral, while undershooting the Fermi target, is already incorporating the dominant lineshape physics. The sub-GeV sub-threshold region is genuinely more complex than either correction alone can describe.

4.3 W -boson mass

Substituting $\Delta\alpha_{\text{had}}^{\text{lm P138}} = 3.718 \times 10^{-3}$ and the P134 leptonic running $\Delta\alpha_{\text{lep}} = 0.031422$ into the Fermi on-shell constraint

$$M_W^2 \left(1 - \frac{M_W^2}{M_Z^2}\right) = \frac{\pi \alpha(M_Z)}{\sqrt{2} G_F}, \quad \alpha(M_Z) = \frac{\alpha}{1 - \Delta\alpha_{\text{lep}} - \Delta\alpha_{\text{had}}^{\text{lm P138}}}, \quad (19)$$

gives

$$M_W^{\text{P138}} = \mathbf{80.383 \text{ GeV}} \quad (+0.008\% \text{ vs PDG } 80.377 \text{ GeV}). \quad (20)$$

This is indistinguishable from the P137 result 80.383 GeV at the precision of the calculation, confirming that the two corrections cancel not just in $\Delta\alpha_{\text{had}}$ but in the derived electroweak observable as well. Table 3 updates the multi-route summary from P137.

Table 3: W -boson mass predictions from TOE-native routes, updated with P138.

Route	M_W (GeV)	Residual vs PDG
P122 (geometric, $\Delta\rho$ from top sector)	80.313	−0.080%
P135 (Fermi, leptonic $\Delta\alpha$ only)	80.445	+0.085%
P136 (Fermi, light mesons NWA)	80.370	−0.009%
P137 (Fermi, light mesons GS)	80.383	+0.008%
P138 (Fermi, GS + FSI + $\rho\omega$)	80.383	+0.008%
PDG	80.377	—

5 Discussion

The main result of this addendum is negative in the gap-closure sense but positive as a structural finding: the two sub-GeV mechanisms identified in P137 do not close the 9.2% gap, and their near-cancellation indicates that the GS lineshape is already near the effective midpoint between “too high” and “too low” sub-threshold contributions. This is consistent with the geometric-mean result of P137: the average of NWA and GS estimates produces 4.107×10^{-3} , within 0.07% of the Fermi target. That near-equality has a deeper interpretation now: the GS integral already implicitly balances the Omnès enhancement against the ρ – ω suppression, because the GS model was fitted to data that include both effects. The separate derivations of this addendum confirm that the two physical mechanisms are of comparable size, enter with opposite signs, and cancel at the precision level relevant to M_W .

The sub-GeV gap of 3.79×10^{-4} therefore survives these two corrections intact. Its origin must lie in spectral content that is not captured by the ρ – ω – ϕ discrete spectrum of $J_3(\mathbb{O})$: specifically, the multi-pion continuum above the two-pion threshold that was identified in P136 as the dominant contributor to the gap between the TOE discrete sum and the PDG dispersive total. A derivation of the $\pi^+\pi^-\pi^0\pi^-\pi^+$ and higher-multiplicity contributions from $J_3(\mathbb{O})$ algebraic structure, or a correction to the heavy-quarkonium f_V^2 values for J/ψ and Υ noted in P136, would be required to close the remaining gap.

6 Open Items

1. Higher-order Omnès corrections. The linear phase-shift approximation $\delta_1^1(s) \approx (s/M_\rho^2) \cdot (\pi/2)$ is valid to roughly 5% accuracy below $\sqrt{s} \approx 0.5 \text{ GeV}$ but breaks down as the ρ resonance is approached. A full dispersive determination of $\delta_1^1(s)$ from $J_3(\mathbb{O})$ pion masses (P113) would improve the FSI estimate. This requires the $I = 1$ P -wave scattering length from the $J_3(\mathbb{O})$ charged-pion sector, which is not yet derived.

2. ε^2 term in the ρ - ω mixing. Equation (9) retained only the linear-in- ε cross-term and dropped the $\varepsilon^2 |B_\omega|^2$ contribution. Near $s = M_\omega^2$, the quadratic term is $\varepsilon^2 \cdot (M_\omega/\Gamma_\omega)^2 \approx (0.04)^2 \cdot (92)^2 \approx 13$, which is large at the ω pole but integrates over the narrow ω width to a contribution of order $\varepsilon^2 \cdot (M_\omega/\Gamma_\omega) \cdot M_\omega^2/M_Z^2 \sim 10^{-6}$ per unit of $|B_\rho|^2$ —negligible compared to the cross-term. The ε^2 term is the ω contribution to the $\pi^+\pi^-$ channel through its admixture of the ρ , which is properly the ω -mediated $\pi\pi$ contribution not counted in the P136/P137 treatment. Its evaluation would require a careful accounting to avoid double-counting with the NWA ω term already present in the P137 total.

3. Coupled-channel lineshape. The full treatment of ρ - ω mixing at the lineshape level requires a 2×2 coupled-channel propagator $\mathbf{D}^{-1}(s) = \text{diag}(D_\rho, D_\omega) - \Pi_{\rho\omega} \mathbf{J}$ where $\Pi_{\rho\omega} = \varepsilon M_\rho^2 \approx -0.0237 \text{ GeV}^2$ is the off-diagonal self-energy derived from P118. This goes beyond the multiplicative correction of equation (8) and would modify the ρ lineshape itself in the region $|s - M_\omega^2| \lesssim \Gamma_\omega^2$. The resulting change in the dispersive integral is of order $|\Pi_{\rho\omega}|^2/(M_\omega^2 - M_\rho^2)^2 \approx (0.024)^2/(0.012)^2 \approx 4$ — but modulated by $\Gamma_\omega/\Gamma_\rho \approx 0.06$, giving an overall effect ~ 0.24 , which is not negligible. A full coupled-channel GS derivation is an open sub-derivation of P138.

4. The multi-pion continuum. Neither mechanism evaluated here contributes to the 3.79×10^{-4} gap. As P136 established, the dominant source of the gap between the TOE discrete resonance sum and the Fermi target is the hadronic continuum from multi-pion production and open-charm threshold, which lies outside the $J_3(\mathbb{O})$ discrete spectrum. The path to closing the gap runs through either a derivation of the four-pion threshold contribution from first principles or a correction to the heavy-quarkonium decay constants for J/ψ and Υ (P136 open item 3), not through further refinement of the sub-GeV ρ - ω sector.