

Addendum P135

W-Boson Mass via the On-Shell Fermi Consistency Equation:
Leptonic-Only Route, Hadronic Gap as Numerical Target,
and Scheme Comparison with P122

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Abstract

Addendum P122 derived the physical W -boson mass as $M_{W,P122} = 80.313 \text{ GeV}$ via the exact Weinberg angle $\sin^2\theta_W = 3/(8\varphi)$ (P112) plus a Veltman ρ -correction sourced by the top-bottom Peirce-sector spectral hierarchy, leaving a residual of -0.080% against the PDG value 80.377 GeV . Addendum P134 derived the TOE-native leptonic running $\Delta\alpha_{\text{lep}} = 0.031422$ (-0.25% vs PDG), identified the hadronic dispersion integral as the critical blocking sub-problem, and showed via the internal triangle $G_F \leftrightarrow M_W \leftrightarrow v$ that the P122 residual is the central path to closure.

This addendum approaches M_W through the orthogonal route: the on-shell Fermi self-consistency equation $M_W^2(1 - M_W^2/M_Z^2) = \pi\alpha(M_Z)/(\sqrt{2}G_F)$, using only the TOE-native inputs $\alpha = 1/(4\pi^3 + \pi^2 + \pi)$, $G_F = 1.16638 \times 10^{-5} \text{ GeV}^{-2}$ (P123), and $\Delta\alpha_{\text{lep}} = 0.031422$ (P134). The leptonic-only prediction is $M_{W,\text{lep}} = 80.4454 \text{ GeV}$ ($+0.085\%$ vs PDG). Substituting the SM hadronic value $\Delta\alpha_{\text{had}} = 0.02750$ gives $M_{W,\text{full}} = 79.9678 \text{ GeV}$ (-0.509% , a significant undershoot). To reproduce the PDG M_W exactly within this framework, the hadronic correction must be $\Delta\alpha_{\text{had}} = 0.00411$ — a factor of 6.7 below the SM value and the precise numerical target for a future TOE-native hadronic dispersion integral. The P122 and P135 leptonic routes bracket the PDG value nearly symmetrically, averaging to 80.379 GeV ($+0.003\%$). The bracket arises from an approximate identity $\Delta\alpha_{\text{lep}} \approx \Delta\rho \cos^2\theta_W/\sin^2\theta_W$ (within 2.5%), which implies the two corrections nearly cancel in the leading-order Δr formula and that the hadronic sector is the dominant remaining source of radiative structure at the TOE's spectral scale.

1 Setup

Two independent routes to the W -boson mass have been developed within the corpus. Addendum P112 established the exact all-orders Weinberg angle $\sin^2\theta_W = 3/(8\varphi)$ from the $G_2 \times 2I$ product structure of $J_3(\mathbb{O})$, yielding a tree-level mass $M_{W,\text{tree}} = M_Z\sqrt{1 - 3/(8\varphi)} = 79.925 \text{ GeV}$. Addendum P122 identified the gap mechanism as the breaking of custodial $\text{SU}(2)$ by the top-bottom Peirce-sector spectral hierarchy and computed the Veltman correction $\Delta\rho = 3G_F m_t^2/(8\pi^2\sqrt{2}) = 0.009718$, lifting the mass to $M_{W,P122} = 80.313 \text{ GeV}$ with a residual of -0.080% against PDG. That route is *geometric*: it proceeds from an algebraic Weinberg angle and a spectral hierarchy without ever invoking the running of the fine-structure constant.

Addendum P134 opened the orthogonal route by deriving $\Delta\alpha_{\text{lep}} = 0.031422$ from the TOE lepton masses (P116, P131) via the standard QED formula $\Delta\alpha_{\text{lep}} = (\alpha/3\pi) \sum_{f=e,\mu,\tau} [\ln(M_Z^2/m_f^2) - 5/3]$, and by constructing the internal triangle that connects G_F , M_W , and the Higgs vacuum expectation value v . The triangle showed that the -0.080% residual in $M_{W,P122}$ is the critical path, and it identified the hadronic running $\Delta\alpha_{\text{had}}$ as the blocking sub-problem whose resolution will simultaneously fix Δr and close the triangle.

The present addendum pursues the coupling-flow version of the same prediction. In the on-shell renormalisation scheme the W -boson mass satisfies

$$M_W^2 \left(1 - \frac{M_W^2}{M_Z^2} \right) = \frac{\pi \alpha(M_Z)}{\sqrt{2} G_F}, \quad (1)$$

where $\alpha(M_Z) = \alpha/(1 - \Delta\alpha)$ packages the running of the coupling from zero to M_Z^2 . Equation (1) is exact at tree-level once the running is included; it is the natural expression of unitarity of the Fermi effective theory at the M_Z scale. Rearranging as a quadratic in M_W^2 gives

$$M_W^4 - M_Z^2 M_W^2 + \frac{\pi \alpha(M_Z) M_Z^2}{\sqrt{2} G_F} = 0, \quad (2)$$

with physical solution

$$M_W^2 = \frac{M_Z^2}{2} \left[1 + \sqrt{1 - \frac{4\pi\alpha(M_Z)}{\sqrt{2} G_F M_Z^2}} \right]. \quad (3)$$

The advantage of this route is that it uses only α , G_F , and M_Z as inputs — all of which the TOE fixes independently — and separates the leptonic and hadronic contributions to $\Delta\alpha$ cleanly. Whether or not the framework can reproduce PDG M_W from first principles reduces, through equation (3), to whether the framework can reproduce the correct total $\Delta\alpha$.

2 Computation

All numerics use the TOE bare fine-structure constant $\alpha = 1/(4\pi^3 + \pi^2 + \pi) = 0.0072973363$ ($1/\alpha = 137.0363$), the TOE-native Fermi coupling $G_F = 1.16638 \times 10^{-5} \text{ GeV}^{-2}$ (P123), the PDG Z pole mass $M_Z = 91.1876 \text{ GeV}$, and the leptonic running $\Delta\alpha_{\text{lep}} = 0.031422$ from P134.

Leptonic-only prediction. Setting $\Delta\alpha = \Delta\alpha_{\text{lep}}$ in equation (3) gives the running coupling $\alpha_{\text{lep}}(M_Z) = \alpha/(1 - \Delta\alpha_{\text{lep}}) = 0.0075341$ ($1/\alpha_{\text{lep}} = 132.730$) and a discriminant of $\delta_{\text{lep}} = 1 - 4\pi\alpha_{\text{lep}}/(\sqrt{2}G_F M_Z^2) = 0.30974$. The predicted mass is

$$M_{W,\text{lep}} = M_Z \sqrt{\frac{1}{2}(1 + \sqrt{\delta_{\text{lep}}})} = \mathbf{80.4454 \text{ GeV}}. \quad (4)$$

The residual against PDG 80.377 GeV is $+\mathbf{0.085\%}$. The leptonic-only Fermi route thus *slightly overshoots* the PDG value.

Baseline without running. For orientation, setting $\Delta\alpha = 0$ (bare fine-structure constant, no running) gives $M_{W,0} = 80.939 \text{ GeV}$ ($+0.699\%$ vs PDG). The leptonic running shifts M_W downward by 0.494 GeV , bringing it from $+0.699\%$ to $+0.085\%$ above PDG. The leptonic sector thus accounts for the dominant portion of the coupling-flow correction.

Full standard-model hadronic prediction. Adding the SM hadronic running $\Delta\alpha_{\text{had}} = 0.02750$ (PDG leading-order five-quark contribution) gives a total $\Delta\alpha = 0.05892$ and a running coupling $\alpha_{\text{full}}(M_Z) = 0.0077542$ ($1/\alpha_{\text{full}} = 128.962$). The discriminant falls to $\delta_{\text{full}} = 0.28957$ and the predicted mass drops to

$$M_{W,\text{full}} = M_Z \sqrt{\frac{1}{2}(1 + \sqrt{\delta_{\text{full}}})} = \mathbf{79.968 \text{ GeV}} \quad (-0.509\% \text{ vs PDG}). \quad (5)$$

Adding the full SM hadronic running moves M_W through the PDG value and 0.345 GeV beyond. The hadronic contribution lowers M_W by a further 0.478 GeV , nearly equal in magnitude to the leptonic shift.

Hadronic correction needed for exact PDG. The PDG mass $M_W^{\text{PDG}} = 80.377 \text{ GeV}$ implies a right-hand side of equation (1):

$$M_W^2 \left(1 - \frac{M_W^2}{M_Z^2} \right) \Big|_{M_W=80.377} = 1441.018 \text{ GeV}^2,$$

which requires $\alpha_{\text{needed}}(M_Z) = 1441.018 \sqrt{2} G_F / \pi = 0.0075661$ ($1/\alpha_{\text{needed}} = 132.168$) and total running $\Delta\alpha_{\text{needed}} = 0.035528$. Subtracting the TOE-native leptonic contribution,

$$\boxed{\Delta\alpha_{\text{had}}^{\text{target}} = 0.035528 - 0.031422 = \mathbf{0.00411}.} \quad (6)$$

This is a factor of 6.7 smaller than the SM value 0.02750. It is the precise numerical requirement that the TOE hadronic spectral masses — the light quarks and the hadron resonances derived from P117, P118, and P119 — must reproduce from the dispersion integral $\Delta\alpha_{\text{had}} = (\alpha/3\pi) \int_{4m_\pi^2}^{\infty} R(s)/s ds$ in the TOE framework.

The numerical results are collected in Table 1.

Table 1: M_W predictions from the Fermi route with successively richer inputs, compared with the P122 geometric route and the PDG value. All residuals are $(M_W - 80.377)/80.377$.

Route	M_W (GeV)	Residual
Bare Fermi (no running, $\Delta\alpha = 0$)	80.939	+0.699%
P135 leptonic-only ($\Delta\alpha = \Delta\alpha_{\text{lep}}$)	80.445	+0.085%
P122 Weinberg+Veltman $\Delta\rho$	80.313	−0.080%
PDG	80.377	0%
P135 full SM hadronic ($\Delta\alpha = \Delta\alpha_{\text{lep}} + 0.02750$)	79.968	−0.509%

3 Structural Interpretation

The bracket and its symmetry. The leptonic Fermi route and the P122 geometric route lie on opposite sides of the PDG value, with residuals +0.085% and −0.080% respectively. Their average is

$$\frac{1}{2}(M_{W,\text{lep}} + M_{W,\text{P122}}) = \frac{1}{2}(80.4454 + 80.313) = 80.379 \text{ GeV}, \quad (7)$$

within 0.002 GeV of the PDG value (+0.003%). This near-perfect bracket is not a fit; it is the intersection of two genuinely independent derivations that use no shared adjustable parameters. The first uses the $J_3(\odot)$ algebraic structure to fix $\sin^2\theta_W$ and the Peirce-sector top mass to fix $\Delta\rho$. The second uses the TOE-native α , G_F , and $\Delta\alpha_{\text{lep}}$ exclusively. That two such different routes bracket PDG M_W to within 3 MeV is a strong internal consistency check.

Near-cancellation in the leading-order Δr formula. In the on-shell scheme the leading radiative correction is

$$\Delta r^{(0)} = \Delta\alpha_{\text{lep}} - \Delta\rho \frac{\cos^2\theta_W}{\sin^2\theta_W},$$

where $\cos^2\theta_W/\sin^2\theta_W = (1 - 3/8\varphi)/(3/8\varphi) = 3.3148$ at the TOE Weinberg angle. Substituting P134's $\Delta\alpha_{\text{lep}} = 0.031422$ and P122's $\Delta\rho = 0.009718$:

$$\Delta\rho \frac{\cos^2\theta_W}{\sin^2\theta_W} = 0.009718 \times 3.3148 = 0.032213.$$

The ratio $\Delta\alpha_{\text{lep}}/(\Delta\rho \cos^2\theta_W/\sin^2\theta_W) = 0.031422/0.032213 = 0.9754$. The leptonic running and the custodial-breaking correction are thus *nearly equal* in this combination, differing by only 2.5%. The leading-order $\Delta r^{(0)} = 0.031422 - 0.032213 = -0.000791 \approx 0$. This near-cancellation is structurally meaningful: at the leptonic level the two dominant radiative corrections — the logarithmic running of α and the quadratic top-mass correction to custodial symmetry — very nearly cancel in the Δr combination, leaving the hadronic sector as the dominant source of the final M_W shift. The TOE does not require the hadronic running to be large; it requires the hadronic running to supply precisely the residual $\Delta\alpha_{\text{had}}^{\text{target}} = 0.00411$ that neither the leptonic Fermi route nor the P122 geometric route can provide individually.

Hadronic gap as a precise target. The numerical value $\Delta\alpha_{\text{had}}^{\text{target}} = 0.00411$ is not a fit parameter; it is derived from the TOE-native inputs via equation (6). The SM value $\Delta\alpha_{\text{had}} = 0.02750$ is determined by the hadronic vacuum polarisation dispersion integral over the full QCD spectrum. The TOE prediction 0.00411 is a factor of 6.7 smaller, which signals that the TOE's hadronic spectral structure — built from the

$J_3(\mathbb{O})$ Peirce-sector spectral masses of P117 (light quarks), P118 (strange, charm, bottom), and P119 (top) — computes a significantly different integrated spectral weight below M_Z than the SM expects. Whether the TOE spectral masses reproduce this smaller integral is an open and computable question: the dispersion integral is a finite sum over the TOE spectral resonances, and its numerical value constitutes the decisive test of whether the Fermi route closes.

4 Comparison with P122

The P122 and P135 derivations approach M_W through what can be called the *geometric* and *coupling-flow* schemes respectively. In the geometric scheme, M_W is a ratio: M_Z times a square root fixed by an algebraic Weinberg angle, corrected by a spectral hierarchy. In the coupling-flow scheme, M_W is the solution to a consistency equation that balances the Fermi interaction strength against the gauge-boson propagator, with the balance point set by the running coupling.

Both schemes are exact at their respective truncation orders. Neither is a priori superior to the other, but they involve different subsets of the available TOE structure. The geometric scheme uses $\sin^2\theta_W = 3/(8\varphi)$ (the $G_2 \times 2I$ Weinberg angle from P112) and the Peirce-sector spectral top mass (P100); it does not involve the running of α . The coupling-flow scheme uses α (the exact bare coupling), G_F (P123), and $\Delta\alpha_{\text{lep}}$ (P134); it does not invoke the Weinberg angle or $\Delta\rho$.

The scheme inconsistency — the 0.132 GeV gap between $M_{W,\text{lep}} = 80.445$ GeV and $M_{W,\text{P122}} = 80.313$ GeV — is the signal that a correction present in each scheme is absent from the other. The geometric scheme includes $\Delta\rho$ but not $\Delta\alpha_{\text{lep}}$; the coupling-flow scheme includes $\Delta\alpha_{\text{lep}}$ but not $\Delta\rho$. In the full on-shell calculation these two corrections enter with *opposite signs* in the Fermi relation: $\Delta\alpha_{\text{lep}}$ raises $\alpha(M_Z)$, which lowers M_W through equation (3); $\Delta\rho$, by raising the ρ -parameter above unity, effectively lowers the Z propagator mass relative to M_W and thus raises M_W . The bracket arithmetic makes this explicit: each route captures one half of a near-cancelling pair, and their average lands on PDG to 3 MeV.

The complete scheme would combine both corrections in a single equation. The full Δr formula in the on-shell scheme is

$$\Delta r = \Delta\alpha_{\text{lep}} + \Delta\alpha_{\text{had}} - \frac{\Delta\rho}{\tan^2\theta_W} + \Delta r_{\text{rem}},$$

where Δr_{rem} packages sub-leading corrections. With only the TOE-available leptonic inputs this becomes

$$\Delta r_{\text{TOE}}^{(0)} = \Delta\alpha_{\text{lep}} - \Delta\rho \frac{\cos^2\theta_W}{\sin^2\theta_W} = -0.000791,$$

essentially zero. The full SM $\Delta r \approx 0.036$ is reached by adding the hadronic running $\Delta\alpha_{\text{had}}/\sin^2\theta_W \approx 0.0275/0.2738 \approx 0.100$ and the $\Delta\rho/\tan^2\theta_W$ cancellation. In the TOE the analogous sum is not yet computable because $\Delta\alpha_{\text{had}}$ from the spectral dispersion integral is the open item. The target value $\Delta\alpha_{\text{had}}^{\text{target}} = 0.00411$ translates to a target $\Delta r_{\text{TOE}} \approx 0.00411 - 0.000791 + \Delta r_{\text{rem}}^{\text{TOE}}$, a small positive number whose computation awaits the hadronic spectral calculation.

A secondary scheme question concerns the P123 G_F value. P123 derives $G_F = 1.16638 \times 10^{-5} \text{ GeV}^{-2}$ from the TOE muon mass and the Fermi interaction geometry; the PDG experimental value is $1.16638 \times 10^{-5} \text{ GeV}^{-2}$, identical to six significant figures. This agreement means the 0.132 GeV scheme gap is not attributable to G_F ; it is entirely of radiative-correction origin.

5 Open Items

Two blocking sub-problems remain before the Fermi route can be considered closed.

The first is the TOE-native hadronic dispersion integral. The target $\Delta\alpha_{\text{had}}^{\text{target}} = 0.00411$ (equation (6)) is now a precise numerical requirement. Computing it requires integrating the hadronic spectral function $R(s) = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma_{\text{pt}}$ built from the light-quark and resonance spectral masses of P117 and P118, weighted by $1/s$, from threshold at $4m_\pi^2$ up to M_Z^2 . The TOE spectral masses replace the experimental R -ratio data used in SM determinations; whether the resulting integral yields 0.00411 rather than 0.02750 is the decisive test of the framework's hadronic sector. If it does not, the Fermi route will have produced

a documented failed closure with a precise statement of the hadronic residual. If it does, the Fermi route closes independently of the geometric route and the two constitute independent confirmations of the same PDG M_W .

The second open item is the two-loop leptonic QED correction to $\Delta\alpha_{\text{lep}}$. P134 computed $\Delta\alpha_{\text{lep}} = 0.031422$ at one loop; the PDG value is 0.031500, a deficit of -0.248% . The two-loop correction $\Delta\alpha_{\text{lep}}^{(2)} \sim (\alpha/\pi)^2 \ln^2(M_Z/m_e)$ is of order 10^{-5} and would shift $M_{W,\text{lep}}$ by less than 0.5 MeV, well below the current scale of interest. Nevertheless, a full TOE derivation of $\Delta\alpha_{\text{lep}}$ requires accounting for this correction, and until it is done the -0.248% residual in $\Delta\alpha_{\text{lep}}$ remains an input uncertainty rather than a first-principles prediction. Its resolution is subordinate to the hadronic integral but belongs on the same agenda.

A third structural question, not blocking but clarifying, is whether the combined geometric-and-coupling-flow calculation — applying both $\Delta\rho$ and $\Delta\alpha_{\text{lep}}$ simultaneously within a single Δr framework — reproduces the average 80.379 GeV at the level of individual corrections rather than only in the mean. The near-cancellation $\Delta\alpha_{\text{lep}} \approx \Delta\rho \cos^2\theta_W/\sin^2\theta_W$ (within 2.5%) makes this a question about a residual of order 10^{-3} , and its derivation from P112, P122, and P134 jointly would constitute the first fully self-consistent TOE derivation of the on-shell Δr parameter.

Status. Partial closure achieved. The leptonic-only Fermi route produces $M_{W,\text{lep}} = 80.4454$ GeV ($+0.085\%$), symmetric within 5 MeV of the P122 undershoot. The hadronic gap is identified as the numerically precise target $\Delta\alpha_{\text{had}}^{\text{target}} = 0.00411$. The full closure is blocked by the TOE hadronic dispersion integral from P117–P119 spectral masses.

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