

Addendum P129

Muon Lifetime from the TOE Fermi Sector:
Closure of Open Item OI-2 of Addendum P123

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Abstract

Addendum P123 derived the Fermi coupling constant $G_F = 1.16638 \times 10^{-5} \text{ GeV}^{-2}$ from the $J_3(\mathbb{O})$ Higgs-sector geometry, leaving as open item OI-2 the muon lifetime τ_μ . This addendum closes OI-2 at leading QED order. Using G_F from P123 and the PDG muon mass $m_\mu = 105.658 \text{ MeV}$, the LO formula gives $\tau_\mu^{\text{LO}} = 2.1873 \times 10^{-6} \text{ s}$ (residual -0.44% vs PDG, as predicted by P123). Applying the leading QED radiative correction — the Kinoshita–Sirlin phase-space factor and the $O(\alpha)$ vertex correction — yields $\tau_\mu^{\text{NLO}} = 2.1945 \times 10^{-6} \text{ s}$ (residual -0.11%). The dominant remaining gap is the $+0.60\%$ residual in the TOE muon mass (Addendum A100 spectral table), which enters τ_μ at fifth power and drives the full-TOE prediction 3.09% below PDG. The two open items are the spectral derivation of m_μ to sub-percent accuracy and the TOE-native derivation of the $O(\alpha)$ vertex correction from the fine-structure constant derived in Papers P01/P36.

1 Setup

The muon lifetime is the first decay observable that assembles two TOE closures simultaneously: the Fermi coupling G_F (P123) and the muon mass m_μ (A100 spectral table). Its leading-order formula in the Fermi effective theory is

$$\tau_\mu^{\text{LO}} = \frac{192\pi^3 \hbar}{G_F^2 m_\mu^5}, \quad (1)$$

with $\hbar = 6.582119 \times 10^{-25} \text{ GeV} \cdot \text{s}$ converting the natural-unit result to seconds. The formula follows from the three-body phase space for $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$ in the limit $m_e/m_\mu \rightarrow 0$ with the full phase-space suppression treated as a radiative correction.

Three TOE-derived inputs enter. The Fermi coupling $G_F = 1.16638 \times 10^{-5} \text{ GeV}^{-2}$ was derived in P123 from the VEV identity $G_F = \sqrt{2} N_{\text{gen}} / (2h^\vee(E_6) m_H^{(0)2})$, where $h^\vee(E_6) = 12$, $N_{\text{gen}} = 3$, and $m_H^{(0)} = 123.11 \text{ GeV}$ (P89); the residual against the PDG value is $+0.00046\%$. The muon mass appears in the A100 spectral table as the second rung of the charged-lepton ladder: $m_\mu = m_e \exp(\mathcal{M}(\pi^2 - \pi))$, where $\mathcal{M} \approx 0.79334$ is the first spectral moment derived from the sector diagonal of $J_3(\mathbb{O})$, giving $m_\mu^{\text{TOE}} = 106.30 \text{ MeV}$ against the PDG value of 105.658 MeV (residual $+0.60\%$). The fine-structure constant $\alpha^{-1} = 4\pi^3 + \pi^2 + \pi \approx 137.036$ enters the QED radiative correction via its TOE identity (P01/P36).

Because $\tau_\mu \propto m_\mu^{-5}$, a $+0.60\%$ error in m_μ maps to a -3.0% error in the predicted lifetime. The spectral closure of m_μ is therefore the controlling open item, discussed further in Section ???. The computation below is therefore organised in two branches: a *main branch* using the TOE G_F with the PDG m_μ , which isolates the contribution of P123 and allows a clean comparison with P123's OI-2 prediction; and a *full-TOE branch* using both TOE inputs, which shows the present state of the programme when all closed results are composed.

2 Computation

2.1 Leading-order result

Inserting $G_F^{\text{TOE}} = 1.16638 \times 10^{-5} \text{ GeV}^{-2}$ and $m_\mu^{\text{PDG}} = 105.658 \text{ MeV}$ into (??) gives

$$\tau_\mu^{\text{LO}}(G_F^{\text{TOE}}, m_\mu^{\text{PDG}}) = 2.1873 \times 10^{-6} \text{ s},$$

a residual of -0.439% against the PDG measured value $\tau_\mu^{\text{PDG}} = 2.196981 \times 10^{-6} \text{ s}$. This agrees with the P123 prediction that OI-2 would find a -0.44% LO gap, consistent with the known QED correction of approximately $+0.42\%$. The GF contribution to the residual is negligible (G_F^{TOE} and G_F^{PDG} agree to 0.00046% , and since $\tau \propto G_F^{-2}$, this shifts the lifetime by only -0.0009%). The full LO residual is driven entirely by the m_μ^{-5} prefactor and the omission of QED corrections.

For reference, the full-TOE branch with $m_\mu^{\text{TOE}} = 106.30 \text{ MeV}$ gives $\tau_\mu^{\text{LOTOE}} = 2.1221 \times 10^{-6} \text{ s}$ (residual -3.41%), confirming that the $+0.60\%$ mass offset dominates with a factor-of-five lever arm on the lifetime.

2.2 Leading QED radiative correction

The exact $O(m_e^2/m_\mu^2)$ phase-space factor is the Kinoshita–Sirlin function

$$f(x) = 1 - 8x^2 + 8x^6 - x^8 - 24x^4 \ln x, \quad x = \frac{m_e}{m_\mu}, \quad (2)$$

which corrects for the finite electron mass in the three-body phase space. With $x = m_e/m_\mu^{\text{PDG}} = 0.51100/105.658 = 4.836 \times 10^{-3}$, one finds $f(x) = 0.999813$ (a -0.019% reduction in the decay rate, i.e. a $+0.019\%$ increase in τ_μ). The leading $O(\alpha)$ vertex correction is

$$\delta_{\text{vert}} = \left(\frac{25}{4} - \frac{\pi^2}{2} \right) \frac{\alpha}{\pi} \approx 1.315 \times \frac{1}{137.036 \pi} \approx 3.056 \times 10^{-3}, \quad (3)$$

which reduces the decay rate by 0.306% . Here α is taken from the TOE identity $\alpha^{-1} = 4\pi^3 + \pi^2 + \pi$, inserting the fundamental structural constant directly.

The combined leading QED factor is

$$\mathcal{F}_{\text{QED}} = f(x) \cdot (1 - \delta_{\text{vert}}) = 0.999813 \times 0.996945 = 0.996759,$$

a net reduction of the decay rate by 0.324% . Since $\tau = 1/\Gamma$, the lifetime increases by $1/\mathcal{F}_{\text{QED}} - 1 = +0.325\%$, which accounts for the bulk of the P123-predicted $+0.42\%$ gap. The residual $\sim 0.10\%$ reflects the higher-order QED terms ($O(\alpha^2)$ and the complete Sirlin electromagnetic correction) that are beyond the scope of this addendum.

2.3 NLO result and residual table

The NLO lifetime is

$$\tau_\mu^{\text{NLO}} = \frac{\tau_\mu^{\text{LO}}}{\mathcal{F}_{\text{QED}}},$$

which increases the predicted value by $+0.325\%$. Table ?? collects both branches.

The main-branch NLO result, -0.11% , represents the predictive precision achievable from the P123 Fermi-sector closure once the leading QED correction is applied. It is consistent with the programme’s current RMS residual of 1.54% (A100) and places the muon lifetime among the more precise TOE predictions once m_μ is stabilised. The full-TOE NLO residual of -3.09% is structurally clean: it equals approximately $-5 \times 0.60\% + 0.04\%$ (round-trip from the mass amplification), so that closing the m_μ spectral prediction to within 0.1% would bring the full-TOE lifetime to within 0.5% of PDG before any QED adjustment.

Table 1: Muon lifetime predictions. Main branch uses TOE G_F (P123) and PDG m_μ . Full-TOE branch uses both TOE inputs (A100 spectral formula for m_μ). QED correction is the leading-order factor $\mathcal{F}_{\text{QED}}$ derived using the TOE α .

Quantity	Value	Residual vs PDG
m_μ^{PDG}	105.658 MeV	—
m_μ^{TOE} (A100 spectral)	106.300 MeV	+0.60%
G_F^{TOE} (P123)	$1.16638 \times 10^{-5} \text{ GeV}^{-2}$	+0.00046%
τ_μ^{LO} (main: TOE G_F + PDG m_μ)	$2.1873 \times 10^{-6} \text{ s}$	−0.44%
τ_μ^{NLO} (main: + leading QED)	$2.1945 \times 10^{-6} \text{ s}$	−0.11%
τ_μ^{LO} (full TOE: both inputs)	$2.1221 \times 10^{-6} \text{ s}$	−3.41%
τ_μ^{NLO} (full TOE: + leading QED)	$2.1290 \times 10^{-6} \text{ s}$	−3.09%
τ_μ^{PDG} (measured)	$2.196981 \times 10^{-6} \text{ s}$	—

3 Open Items

OI-1 (P129): Spectral derivation of m_μ to sub-percent accuracy. The A100 spectral ladder gives $m_\mu^{\text{TOE}} = 106.30 \text{ MeV}$ from the formula $m_\mu = m_e \exp(\mathcal{M}(\pi^2 - \pi))$, with the first spectral moment $\mathcal{M} \approx 0.79334$ derived from the $J_3(\mathbb{O})$ sector diagonal. The +0.60% residual against PDG enters τ_μ with a factor-of-five amplification and is the dominant source of error in the full-TOE prediction. The open item is to derive $\mathcal{M}$ from the Peirce decomposition with sufficient precision that the mass residual falls below 0.1%, at which point the mass contribution to the lifetime residual drops below 0.5% and the QED correction becomes the controlling term. This is distinct from the question of whether the spectral formula itself is exact at all orders; it is specifically a question of precision in the derivation of $\mathcal{M}$.

OI-2 (P129): TOE-native derivation of the $O(\alpha)$ vertex correction. The vertex correction (??) was applied using the PDG phase-space function and the standard QED calculation, but with α replaced by the TOE value $\alpha^{-1} = 4\pi^3 + \pi^2 + \pi$. The structure of the coefficient $25/4 - \pi^2/2 = 1.315$ has not been derived from the $J_3(\mathbb{O})$ geometry: it arises in conventional QED from the scalar integral over the electron propagator in the one-loop W -vertex diagram. A TOE-native derivation would express this coefficient in terms of the Peirce sector structure or the G_2 orbit data, connecting the electroweak vertex correction to the same algebraic source as α itself. The phase-space factor $f(x)$ contributes only −0.019% and is negligible at current precision; the vertex term at −0.306% is the piece that requires derivation.

Status of OI-2 from P123. Open item OI-2 of Addendum P123 asked for the derivation of τ_μ ; that item is now closed at LO and at leading QED order. The main-branch NLO prediction, $\tau_\mu^{\text{NLO}} = 2.1945 \times 10^{-6} \text{ s}$ (residual −0.11%), is consistent with the P123 statement that the LO gap of −0.44% is covered by known QED corrections. The residual −0.11% after the leading QED correction is attributed to higher-order terms ($O(\alpha^2)$, hadronic radiative corrections) that are beyond the programme’s current scope. The two new open items registered above (spectral m_μ precision and TOE-native vertex derivation) are the natural successors within the weak-decay branch of the programme.

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