

Addendum P120: m_u Quark Mass from Confinement Scale

G₂ orbit prediction within experimental 1σ ; J₁ confinement formula open

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Abstract

P114 derived the light quark masses from G₂ orbit assignments (P83) and a Peirce QCD correction $\delta = (8/9)\alpha_s(M_Z)/\pi$. This correction closes m_d (+0.16%) and m_s (−0.10%) but leaves m_u at 2.329 MeV (+7.83% above the PDG central value 2.16 MeV). P114 flagged the residual as an open item, noting proximity to the confinement scale $m_{\text{conf}} = \pi \cdot \text{MU} \cdot m_e = 1.273$ MeV.

Here we carry out the systematic closure attempt for m_u . The outcome is a documented open item, not a failure: both the bare G₂ orbit (2.410 MeV) and the Peirce-corrected value (2.329 MeV) lie within the PDG 1σ band [1.90, 2.65] MeV. The orbit prediction is experimentally consistent. What is lacking is a first-principles derivation that pins the prediction to the confinement scale precisely. We name the blocking sub-problem: a J₁ fold-map self-energy formula (the J₁ analogue of the P₁₃ formula that fixed m_τ in P116).

Contents

1 Setup

1.1 Anchor: G₂ orbit and Peirce correction (P83, P114)

The G₂ orbit masses for light quarks are (P83):

$$m_u^{\text{orbit}} = 2.410 \text{ MeV}, \quad (1)$$

$$m_d^{\text{orbit}} = 4.840 \text{ MeV}, \quad (2)$$

$$m_s^{\text{orbit}} = 98.200 \text{ MeV}. \quad (3)$$

The Peirce QCD correction from P114:

$$\delta = \frac{d_P}{h^\vee(F_4)} \cdot \frac{\alpha_s(M_Z)}{\pi} = \frac{8}{9} \cdot \frac{0.1186}{\pi} = 3.356\%, \quad (4)$$

where $d_P = 8$ is the Peirce block dimension and $h^\vee(F_4) = 9$ is the dual Coxeter number of F_4 . Applied to the d and s orbits, this gives (P114):

$$m_d = 4.840 \times (1 - 0.03356) = 4.677 \text{ MeV} \quad (+0.16\% \text{ vs PDG}), \quad (5)$$

$$m_s = 98.200 \times (1 - 0.03356) = 94.905 \text{ MeV} \quad (-0.10\% \text{ vs PDG}). \quad (6)$$

1.2 Baseline for m_u

Applied to the u orbit:

$$m_u^{\text{QCD}} = 2.410 \times (1 - 0.03356) = 2.329 \text{ MeV}. \quad (7)$$

PDG values (2025):

Quantity	Value	Source
m_u central	2.16 MeV	PDG 2025
m_u 1σ band	[1.90, 2.65] MeV	PDG
m_u^{orbit}	2.410 MeV	P83 G_2 orbit
m_u^{QCD}	2.329 MeV	P114 Peirce correction
Gap (orbit vs PDG central)	+11.6%	
Gap (QCD vs PDG central)	+7.83%	
Orbit within 1σ ?	Yes	[1.90, 2.65]
QCD within 1σ ?	Yes	[1.90, 2.65]

The confinement scale (P114):

$$m_{\text{conf}} = \pi \cdot \text{MU} \cdot m_e = \pi \times 0.79334 \times 0.51100 \text{ MeV} = 1.273 \text{ MeV}. \quad (8)$$

This is below the PDG band — it is a scale, not a direct prediction for m_u . The ratio:

$$\frac{m_u^{\text{orbit}}}{m_{\text{conf}}} = \frac{2.410}{1.273} = 1.893. \quad (9)$$

2 Closure attempt

2.1 Loop 1 — Peirce QCD correction

Mechanism. Apply the same Peirce correction that closes m_d and m_s .

Computation. $m_u^{\text{QCD}} = 2.329 \text{ MeV}$ (above). Residual from PDG central: +7.83%.

Decision. The residual shrank from +11.6% to +7.83%. Both values are within the PDG 1σ band. The mechanism is partially operative, but the residual is at the scale of experimental uncertainty. Loop continues with new baseline +7.83% from PDG central.

2.2 Loop 2 — J_1 confinement-scale structure

Mechanism candidate. The proximity of m_u to m_{conf} (ratio ≈ 1.89 , eq. ??) suggests a non-perturbative J_1 confinement correction analogous to the Peirce fold-map self-energy that fixed m_τ in P116. P116 derived:

$$m_\tau/m_e = \exp(\text{MU} \cdot (E_{\text{self}}(\lambda_\tau) - \pi)), \quad (10)$$

from the P_{13} Peirce block self-energy (off-diagonal J_1 – J_3 coupling). For m_u , the relevant object would be an *intra-sector* J_1 fold-map formula, since u is a J_1 quark with no inter-sector block connection to J_3 .

Computation. The ratio $m_u^{\text{orbit}}/m_{\text{conf}} = 1.893$ was tested against all TOE-native dimensionless quantities at the relevant scale:

Candidate ratio	Value	Deviation from 1.893
2	2.000	+5.7%
1 + MU	1.793	−5.3%
1/MU ²	1.589	−16.0%
$\phi^2 = (1 + \sqrt{5})^2/4$	2.618	+38%
e	2.718	+44%
π	3.142	+66%
1 + $\pi\alpha$	1.023	−46%
1 + $\alpha_s/3$	1.040	−45%

No TOE-native ratio matches $m_u^{\text{orbit}}/m_{\text{conf}} = 1.893$ within the closure threshold ($< 1\%$ at this precision). The nearest candidate is $1 + \text{MU} = 1.793$, which is 5.3% off — a discrepancy larger than the gap we are trying to close.

Decision. The mechanism is structurally motivated but cannot be completed without the missing J_1 fold-map formula. There is no closed-form expression in the current framework that produces m_u from m_{conf} exactly. All plausible structural elements have been exhausted. Exit loop; proceed to Step 5 (documented open closure).

3 Status theorem and blocking sub-problem

Theorem 1 (m_u consistency). *Both the G_2 orbit prediction $m_u^{\text{orbit}} = 2.410 \text{ MeV}$ and the Peirce-corrected value $m_u^{\text{QCD}} = 2.329 \text{ MeV}$ are consistent with the experimental value $m_u = 2.16^{+0.49}_{-0.26} \text{ MeV}$ (PDG 2025) at the 1σ level. The orbit prediction is not falsified by current data.*

Remark 1. m_u^{QCD} is 0.34σ above the PDG central value. The full PDG 1σ band spans $[1.90, 2.65] \text{ MeV}$, a range of 0.75 MeV , reflecting the current difficulty of determining light quark masses from lattice QCD at physical pion masses. Any prediction within this band is experimentally consistent.

Theorem 2 (Blocking sub-problem). *A first-principles derivation of m_u that pins the prediction below the 1σ band width requires a J_1 fold-map self-energy formula*

$$m_u = m_e \cdot F_1(\text{MU}, \alpha, \alpha_s), \quad (11)$$

where F_1 encodes the intra-sector J_1 Peirce geometry analogously to how m_τ was derived from the P_{13} inter-sector formula in P116.

The distinction from P116:

- P116 used the *off-diagonal* Peirce block P_{13} (J_1 – J_3 coupling), which is geometrically constrained by the G_2 -symmetric boundary of $J_3(\mathbb{O})$.
- The J_1 fold-map would be an *intra-sector* formula. The J_1 block has dimension 1 (it is the diagonal entry $J_1 = \text{diag}(1, 0, 0)$ in the Peirce decomposition), and its self-energy must be computed from the J_1 – J_1 propagator, which is not yet derived in the framework.

4 Why m_u behaves differently from m_d and m_s

The Peirce correction closes m_d and m_s because these quarks sit in the J_1 off-diagonal slots, where the QCD correction at scale M_Z is the leading perturbative effect. For m_u (the lightest quark), two additional non-perturbative effects become relevant that the Peirce correction does not capture:

1. Proximity to m_{conf} . $m_u \approx 1.9 \cdot m_{\text{conf}}$, meaning m_u is not deep in the perturbative regime. The Peirce correction was derived from perturbative QCD (the α_s/π expansion). At masses this close to m_{conf} , non-perturbative J_1 contributions from the confining background are expected to be significant.

2. Isospin breaking. m_u and m_d are not degenerate ($m_d/m_u \approx 2.24$ in the PDG). The G_2 orbit assigns them to different nodes of the same J_1 block. The isospin-breaking correction (which sets $m_d \neq m_u$ in the orbit) comes from the EM sector (the u quark has $+2/3$ charge vs d quark $-1/3$). This correction is of order $\alpha \cdot m_u$, but the precise geometric factor has not been derived.

Both effects — non-perturbative confinement corrections and isospin-breaking EM contributions — require structural input that is not yet in the framework. The Peirce correction is the perturbative skeleton; m_u is where the skeleton does not yet have enough flesh.

5 What would close it

Two derivations would complete the m_u sector:

Path A — J₁ fold-map self-energy. Derive the intra-sector J₁ self-energy formula analogously to P116. P116 uses the spectral density anchor $\mu_0 = 4\pi^3 + \pi^2 + \pi$ to set the exponent of the fold-map for the P₁₃ block. The J₁ analogue would use the same spectral density but with J₁-sector boundary conditions. Concretely: compute $E_{\text{self}}(\lambda_u)$ for the J₁ quark sector and find whether $m_u = m_e \cdot \exp(f(\text{MU}, \lambda_u))$ closes against the PDG central value.

Path B — EM isospin correction. The u - d mass split is $m_d - m_u \approx 2.52 \text{ MeV}$ (PDG central). The orbit split is $m_d^{\text{orbit}} - m_u^{\text{orbit}} = 4.840 - 2.410 = 2.430 \text{ MeV}$, which already matches PDG to within the experimental band. An EM correction of order $\alpha \cdot m_N \sim 1.3 \text{ MeV}$ (nucleon-scale) could adjust both m_u and m_d while preserving their difference. Deriving the exact EM factor from J₁ charge structure would close both simultaneously.

6 Summary

Quantity	Value	Status
m_u^{orbit} (P83)	2.410 MeV	within PDG 1σ
m_u^{QCD} (P114)	2.329 MeV	within PDG 1σ , 0.34σ above central
m_u PDG central	2.16 MeV	ground truth
m_u PDG 1σ	[1.90, 2.65] MeV	
Residual (QCD vs central)	+7.83%	within experimental uncertainty
Closure attempt	Exhausted	Peirce (partial) + confinement (no formula)
Blocking sub-problem	J ₁ fold-map	Theorem ??

The G₂ orbit prediction for m_u is experimentally consistent. The framework predicts the right order and the right 1σ band; it does not yet pin the central value. This is not a falsification — it is a precision frontier. The next derivation that matters for m_u is the J₁ intra-sector fold-map, which will require extending the spectral self-energy formalism of P116 from the off-diagonal to the diagonal Peirce block.