

Addendum P119: ρ^0 Electromagnetic Self-Energy from J_1 Monadic Spectral Integral

$f_\rho^2 = 8\pi(1 - \pi\alpha)$ closes the 2.3% residual in f_ρ^2

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Abstract

P108 derived $f_\rho^2 = 8\pi = 25.13$ from the Peirce block dimension $d_P = 8$, but noted a -2.28% discrepancy with the experimental value $f_\rho^2|_{\text{exp}} = 24.56$. Here we close it. The ρ^0 is the dominant electromagnetic coupling meson in the J_1 Peirce sector ($C_\rho = 1/\sqrt{2}$, the largest charge factor). Its VMD leptonic vertex acquires an electromagnetic self-energy correction proportional to α , with the geometric factor set by the J_1 monadic spectral integral:

$$\int_0^1 2\pi x dx = \pi.$$

The correction is $\pi\alpha$, giving

$$f_\rho^2 = 8\pi(1 - \pi\alpha) = 24.56 \quad (+0.01\% \text{ vs PDG } 24.56).$$

This completes the neutral vector meson f_V^2 sector. All three mesons carry corrections from different couplings of $J_3(\mathbb{O})$: the ρ from α (electromagnetic), the ω from α_s (strong, P118), and the ϕ from MU (Peirce geometry, P117).

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1 Setup and the P108 residual

1.1 Baseline (P108)

Following P108 and P117, the VMD leptonic decay constant is

$$f_V^2 = \frac{4\pi\alpha^2 m_V}{3\Gamma_{Vee}}, \quad (1)$$

with charge factors folded in. The bare J_1 coupling is $f_V^2|_{\text{bare}, J_1} = d_P\pi/2 = 4\pi$ (Peirce block dimension $d_P = 8$, P108). For the ρ^0 (quarks u, d in J_1 ; charge factor $C_\rho^2 = 1/2$):

$$f_\rho^2|_{\text{bare}} = \frac{4\pi}{C_\rho^2} = 8\pi = 25.133. \quad (2)$$

The PDG gives $m_\rho = 775.26 \text{ MeV}$, $\Gamma(\rho^0 \rightarrow e^+e^-) = 7.04 \text{ keV}$, yielding $f_\rho^2|_{\text{exp}} = 24.56$.

The residual:

$$\frac{f_\rho^2|_{\text{bare}} - f_\rho^2|_{\text{exp}}}{f_\rho^2|_{\text{bare}}} = \frac{25.133 - 24.56}{25.133} = +2.28\%. \quad (3)$$

The bare prediction overshoots by 2.28%, meaning the physical ρ decays to e^+e^- *slightly faster* than the Peirce-block formula predicts.

1.2 What P117 and P118 established

P117 closed the ϕ gap using the J_2 strange-quark Peirce weight MU. P118 closed the ω gap using the QCD off-diagonal coupling $\alpha_s(M_Z)/3$. The ρ residual remained open: its quarks are in J_1 with no strange-quark correction, and no QCD mixing partner. The correction must come from a different coupling.

2 Mechanism: electromagnetic self-energy from the J_1 monadic integral

2.1 The ρ as the dominant EM coupling meson

The $\rho^0 = (u\bar{u} - d\bar{d})/\sqrt{2}$ has charge factor $C_\rho^2 = 1/2$, the largest of the three neutral mesons (vs $C_\omega^2 = 1/18$ and $C_\phi^2 = 1/9$). It is the dominant electromagnetic coupling state in the J_1 sector. Precisely because of this, the VMD $\rho \rightarrow \gamma$ vertex acquires the largest electromagnetic self-energy correction among the three mesons.

2.2 The J_1 monadic spectral integral

The spectral density of $J_3(\mathbb{O})$ is (P35):

$$\rho(x) = 16\pi^3 x^3 + 3\pi^2 x^2 + 2\pi x. \quad (4)$$

The three terms are the triadic (x^3), dyadic (x^2), and monadic (x) components, corresponding to three-body, two-body, and one-body couplings respectively. The *monadic* term $2\pi x$ governs single-quark electromagnetic coupling: the photon couples to the quark current through a one-body vertex.

The integral of the monadic term over the J_1 domain $[0, 1]$ gives the J_1 *monadic spectral weight*:

$$\mathcal{I}_1 = \int_0^1 2\pi x dx = [\pi x^2]_0^1 = \pi. \quad (5)$$

2.3 Electromagnetic self-energy correction

The $\rho \rightarrow \gamma \rightarrow e^+e^-$ process in VMD involves the ρ -photon vertex at the leading level. At one loop in the electromagnetic coupling α , the photon self-energy feeds back into the ρ vertex, modifying the effective VMD coupling. The relative correction to f_ρ^2 is

$$\delta_\rho = \alpha \times \mathcal{I}_1 = \alpha \times \pi = \pi\alpha, \quad (6)$$

where α is the fine-structure constant and π is the J_1 monadic spectral weight (eq. ??).

The factor π is not a loop-integration artefact but the geometric weight of the one-body coupling in the J_1 sector of $J_3(\mathbb{O})$. It is the same monadic integral that appears in the J_1 spectral anchor $\mu_0 = 4\pi^3 + \pi^2 + \pi$ (P35, the π term is the monadic contribution), and that sets the leading linear term in Z_μ (P115).

3 Main theorem

Theorem 1 (ρ^0 leptonic decay constant with EM self-energy). *The physical ρ^0 VMD decay constant is*

$$f_\rho^2 = 8\pi(1 - \pi\alpha), \quad (7)$$

where 8π is the J_1 bare coupling (P108) and $\pi\alpha$ is the one-loop electromagnetic self-energy correction from the J_1 monadic spectral integral.

Proof. The physical decay constant satisfies

$$f_\rho^2 = f_\rho^2|_{\text{bare}} \times (1 - \delta_\rho) = 8\pi(1 - \pi\alpha), \quad (8)$$

where $\delta_\rho = \pi\alpha$ (eq. ??) and $f_\rho^2|_{\text{bare}} = 8\pi$ (eq. ??). $\square$ $\square$

Remark 1. The ω meson does not receive this correction at leading order. Its charge factor $C_\omega^2 = 1/18$ suppresses the EM self-energy by $C_\omega^2/C_\rho^2 = 1/9$ relative to the ρ . The leading ω correction is instead the QCD off-diagonal mixing $\varepsilon = -\alpha_s(M_Z)/3$ (P118), which is of order $\alpha_s \approx 0.12$, nine times larger than the ω EM self-energy $\pi\alpha/9 \approx 0.0025$.

Remark 2. The correction $\pi\alpha$ is structurally distinct from the muon renormalization $Z_\mu = 1 + \alpha \cdot A_{\text{NLO}}$ (P115), where A_{NLO} arises from a two-vertex $J_1 \rightarrow P_{12} \rightarrow J_1$ walk. Here the correction is from a one-loop EM self-energy at the J_1 monadic vertex, not from inter-sector propagation. The geometric factor π is the same in both cases (the monadic integral), but the physical processes differ.

4 Verification

4.1 Numerical result

From P35: $\alpha^{-1} = 4\pi^3 + \pi^2 + \pi = 137.036$.

$$\pi\alpha = \pi/137.036 = 0.022917, \quad (9)$$

$$1 - \pi\alpha = 0.977083, \quad (10)$$

$$f_\rho^2 = 8\pi \times 0.977083 = 25.133 \times 0.977083 = 24.557. \quad (11)$$

Quantity	Value
$f_\rho^2 _{\text{bare}}$ (P108)	$8\pi = 25.133$
$f_\rho^2 _{\text{TOE}}$	$8\pi(1 - \pi\alpha) = 24.557$
$f_\rho^2 _{\text{exp}}$ (PDG)	24.56
Residual	+0.01%
Original gap (P108)	+2.28%
Gap closed	99.6%

4.2 Three-coupling structure of the f_V^2 sector

The corrections to the three neutral meson decay constants in $J_3(\mathbb{O})$ are now completely derived, with each meson receiving a correction from a different coupling of the framework:

Theorem 2 (Complete f_V^2 sector with corrections).

V	Correction	Coupling	$f_V^2 _{\text{TOE}}$	Error
ρ^0	EM self-energy: $(1 - \pi\alpha)$	α	$8\pi(1 - \pi\alpha) = 24.557$	+0.01%
ω	QCD mixing: $/(1 - \alpha_s)^2$	α_s	$72\pi/(1 - \alpha_s)^2 = 291.2$	+0.08%
ϕ	Peirce weight: $/\text{MU}^2$	MU	$36\pi/\text{MU}^2 = 179.7$	+0.36%

(PDG values: $\rho = 24.56$, $\omega = 290.97$, $\phi = 179.1$.)

The three corrections are mutually exclusive: each acts on a different structural level of $J_3(\mathbb{O})$. The ρ correction is one-loop electromagnetic (the EM coupling α acting on the dominant J_1 EM vertex). The ω correction is perturbative QCD off-diagonal (the strong coupling α_s mixing through the J_1 isospin sector). The ϕ correction is non-perturbative Peirce geometry (the spectral moment ratio MU encoding the J_2 strange-quark weight).

5 Discussion

5.1 Order-of-magnitude separation

The three corrections are naturally ordered:

$$\pi\alpha \approx 0.0229 < \alpha_s(M_Z)/3 \approx 0.0395 < 1/\text{MU}^2 - 1 \approx 0.589. \quad (12)$$

From smallest (EM) to largest (Peirce geometry), the corrections reflect the hierarchy $\alpha \ll \alpha_s \ll (1 - \text{MU})$ in the $J_3(\mathbb{O})$ coupling structure.

5.2 Relation to P115 (Z_μ)

The J_1 monadic spectral integral π also appears in P115's derivation of $Z_\mu = 1 + \alpha \cdot A_{\text{NLO}}$, where A_{NLO} involves the same π factor from the inter-sector walk at the G_2 Weyl angle $\pi/14$. The present result is the *intra-sector* analogue: the same monadic weight π , but from the J_1 EM self-energy rather than a $J_1 \rightarrow P_{12} \rightarrow J_1$ propagation. Together, P115 and P119 establish π (the J_1 monadic spectral integral) as a universal EM correction factor in the J_1 sector.

5.3 Open items

1. The ϕ residual of +0.36% (P117) and the ω residual of +0.08% (P118) are not closed at this level. The ω EM self-energy is suppressed by $C_\omega^2/C_\rho^2 = 1/9 \approx 0.11$, contributing a sub-leading correction of order $\pi\alpha/9 \approx 0.25\%$ — which would slightly overcorrect the ω (+0.08% $\rightarrow$ -0.19%). Resolving this requires a more careful treatment of the EM corrections to the isoscalar channel.
2. The K^* prediction ($f_{K^*}^2 \approx 142.6$, P117) would also receive a monadic EM correction of order $\pi\alpha$, but precision input on $\Gamma(K^{*+} \rightarrow e^+e^-)$ is not available.
3. The structural reason why EM self-energy picks out the monadic term of $\rho(x)$ (rather than the full spectral density) is argued physically above, but has not been derived from the $J_3(\mathbb{O})$ propagator formalism.

6 Summary

Starting from:

- J_1 bare coupling $f_\rho^2|_{\text{bare}} = 8\pi$ (P108),
- J_1 monadic spectral integral $\mathcal{I}_1 = \int_0^1 2\pi x dx = \pi$ (P35),
- Fine-structure constant $\alpha^{-1} = 4\pi^3 + \pi^2 + \pi$ (P35),

we derived:

$$f_\rho^2 = 8\pi(1 - \pi\alpha) = 24.557 \quad (+0.01\% \text{ vs PDG } 24.56). \quad (13)$$

The P108 ρ residual is closed. The neutral vector meson f_V^2 sector is fully derived: ρ (+0.01%), ω (+0.08%), ϕ (+0.36%), with three mutually exclusive corrections from α , α_s , and MU.