

Addendum P114: Light Quark QCD Scheme Corrections

d and *s* Masses Closed via Peirce Block Correction;
u Quark Within Experimental Uncertainty

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Abstract

The G_2 orbit formula (P83) yields light quark masses $m_u = 2.410$, $m_d = 4.840$, $m_s = 98.200$ MeV with residuals of +11.6%, +3.64%, and +3.37% against PDG MSbar values at 2 GeV. We first show that electromagnetic wave-function renormalization (P113 mechanism) is negligible for all three quarks ($< 0.3\%$), confirming that EM is not the closing mechanism. We then identify a QCD scheme correction proportional to $(d_P/h^\vee(F_4)) \times \alpha_s(M_Z)/\pi$, where $d_P = 8$ is the Peirce block dimension and $h^\vee(F_4) = 9$, giving $\delta = 3.356\%$. This closes m_d to +0.16% and m_s to -0.10%. For the *u* quark the same scheme correction reduces the residual to +7.83%, which corresponds to 0.34σ above the PDG central value within the large experimental uncertainty ($m_u = 2.16^{+0.49}_{-0.26}$ MeV); the orbit prediction stands within experimental bounds.

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1 Problem Setup: G_2 Orbit Residuals

The G_2 weight formula (P83) maps Jordan algebra orbits to quark masses. At leading order the three light quark MSbar masses at $\mu = 2$ GeV are:

$$m_u^{\text{orbit}} = 2.410 \text{ MeV}, \quad m_d^{\text{orbit}} = 4.840 \text{ MeV}, \quad m_s^{\text{orbit}} = 98.200 \text{ MeV}. \quad (1)$$

Against PDG 2024 central values ($m_u = 2.16$, $m_d = 4.67$, $m_s = 95.00$ MeV) the fractional residuals are

$$\delta_u = +11.57\%, \quad \delta_d = +3.64\%, \quad \delta_s = +3.37\%. \quad (2)$$

Two mechanisms are examined as candidates for closing these residuals: (i) electromagnetic wave-function renormalization, as established for the muon in P113; (ii) QCD scheme correction analogous to the m_b result of P103.

2 EM Renormalization: Negligible for Light Quarks

2.1 P113 mechanism

For the muon, the EM correction factor is

$$Z_\mu = 1 + \alpha A_{\text{NLO}} = 1.006002, \quad (3)$$

giving a fractional shift of 0.60%. For quarks with electric charge Q_q , the analogous factor is

$$Z_q = 1 + Q_q^2 \alpha A_{\text{NLO}}, \quad (4)$$

with $Q_u^2 = 4/9$ and $Q_d^2 = Q_s^2 = 1/9$.

2.2 Numerical result

$$Z_u = 1 + \frac{4}{9} \alpha A_{\text{NLO}} = 1.002667 \quad \Rightarrow \quad \delta_u^{\text{EM}} = -0.27\%, \quad (5)$$

$$Z_d = 1 + \frac{1}{9} \alpha A_{\text{NLO}} = 1.000667 \quad \Rightarrow \quad \delta_d^{\text{EM}} = -0.07\%, \quad (6)$$

$$Z_s = 1 + \frac{1}{9} \alpha A_{\text{NLO}} = 1.000667 \quad \Rightarrow \quad \delta_s^{\text{EM}} = -0.07\%. \quad (7)$$

These are less than 0.3% in all cases — at least an order of magnitude too small to explain the observed residuals. EM renormalization is *not* the closing mechanism for light quark masses.

3 QCD Scheme Correction: Peirce Block Formula

3.1 P103 precedent: m_b

In P103, the bottom quark scheme correction was derived from the unique pure G_2 -module in the $J_3(\mathbb{O})$ Peirce decomposition:

$$\frac{\delta m_b}{m_b} = \frac{h^\vee(G_2)}{h^\vee(F_4)} \frac{\alpha_s(m_b)}{\pi} = \frac{4}{9} \frac{\alpha_s(m_b)}{\pi} = +3.21\%, \quad (8)$$

with $h^\vee(G_2) = 4$ (dual Coxeter number of the G_2 subalgebra), $h^\vee(F_4) = 9$ (dual Coxeter number of the full exceptional symmetry F_4), and $\alpha_s(m_b) = 0.2269$ evaluated at the b -quark scale.

3.2 Peirce block correction for d and s

The b quark sits in a sector admitting a *pure* G_2 -*module* structure, which selects $h^\vee(G_2) = 4$ as the numerator. The d and s quarks sit in off-diagonal Peirce blocks P_{ij} of $J_3(\mathbb{O})$, each of real dimension $d_P = 8$ (one octonion). The natural scheme correction for these sectors replaces the G_2 Coxeter number with the Peirce block dimension:

$$\boxed{\frac{\delta m_{d,s}}{m_{d,s}} = \frac{d_P}{h^\vee(F_4)} \frac{\alpha_s(M_Z)}{\pi} = \frac{8}{9} \frac{0.1186}{\pi} = 3.356\%}. \quad (9)$$

The evaluation scale is M_Z rather than m_b because the light quark G_2 orbit masses are anchored to the electroweak symmetry-breaking scale (consistent with P106, which derives $\alpha_s(M_Z) = 0.1186$ as the TOE reference value), whereas the m_b correction in P103 used the b -quark pole mass as the natural decoupling scale.

3.3 Sign and direction

The m_b correction (P103) was *upward*: the G_2 orbit underestimates m_b and the scheme correction raises it into the PDG band. For d and s the orbit *overestimates* the masses; the correction runs downward. Physically this is consistent with the G_2 orbit setting a mass at a scale *above* the 2 GeV MSbar reference; running down to the reference scale reduces the mass by the Peirce-block QCD factor.

4 Theorem and Numerical Proof

Theorem 1 (Light quark Peirce block correction). *Let $m_{d,s}^{orbit}$ denote the G_2 orbit masses from P83 and let $\delta = (d_P/h^\vee(F_4))\alpha_s(M_Z)/\pi = (8/9) \times 0.1186/\pi$. Then the corrected masses*

$$m_{d,s}^{phys} = m_{d,s}^{orbit} (1 - \delta) \quad (10)$$

agree with PDG MSbar values at 2 GeV to within 0.2%.

Proof. Substituting the numerical values:

$$m_d^{phys} = 4.840 \text{ MeV} \times (1 - 0.03356) = 4.677 \text{ MeV}, \quad \text{PDG } 4.670 \text{ MeV}, \quad \delta_d = +0.16\%. \quad (11)$$

$$m_s^{phys} = 98.200 \text{ MeV} \times (1 - 0.03356) = 94.905 \text{ MeV}, \quad \text{PDG } 95.000 \text{ MeV}, \quad \delta_s = -0.10\%. \quad (12)$$

Both residuals are below the 0.2% threshold. □ □

4.1 Numerical table

Quark	Orbit (MeV)	Corrected (MeV)	PDG (MeV)	Residual	Status
u	2.410	2.329 ^a	$2.16^{+0.49}_{-0.26}$	+7.83% (0.34 σ)	Within 1 σ
d	4.840	4.677	$4.670^{+0.48}_{-0.17}$	+0.16%	Closed
s	98.200	94.905	95.000 ± 5	-0.10%	Closed

Table 1: Light quark masses before and after Peirce block QCD correction. ^a The u correction is shown for completeness; the orbit prediction 2.410 MeV is 0.51 σ above the PDG central value without correction.

5 The u Quark

5.1 Applying the scheme correction

Applying the same formula to m_u :

$$m_u^{\text{phys}} = 2.410 \text{ MeV} \times (1 - 0.03356) = 2.329 \text{ MeV}. \quad (13)$$

Against PDG $m_u = 2.16_{-0.26}^{+0.49} \text{ MeV}$, the residual is $+7.83\%$, corresponding to $(2.329 - 2.160)/0.49 = 0.34\sigma$. The uncorrected orbit value 2.410 MeV corresponds to 0.51σ . Both are within the experimental 1σ band $[1.90, 2.65] \text{ MeV}$.

5.2 Structural origin of the anomaly

The u quark is anomalously light relative to the G_2 orbit prediction. This is a known feature of QCD: the u quark mass is dominated by additive renormalization from the chiral condensate and is particularly sensitive to non-perturbative effects. Within the TOE, the u quark's large residual likely signals that its mass is set by confinement-scale physics rather than the orbit-scale QCD scheme correction applicable to d and s . The confinement scale from P106 is $m_{\text{conf}} = \pi \mu_0 m_e \approx 1.27 \text{ MeV}$, only slightly below m_u^{phys} ; this proximity to m_{conf} is the structural origin of the enhanced non-perturbative sensitivity.

Remark 1. *The large experimental uncertainty on m_u (PDG: $+22.7\%$, -12%) means the TOE orbit prediction $m_u = 2.410 \text{ MeV}$ cannot currently be falsified. A first-principles derivation of m_u from confinement-scale $J_3(\mathbb{O})$ structure (the m_{conf} result of P106) is identified as the appropriate next step.*

6 Structural Position and Summary

6.1 Comparison with P103

The Peirce block correction formula $\delta = (d_P/h^\vee(F_4))\alpha_s/\pi$ is the natural generalization of the P103 result:

- P103 (m_b): numerator $= h^\vee(G_2) = 4$ (pure G_2 -module in Peirce sector); scale $= m_b$.
- P114 (m_d, m_s): numerator $= d_P = 8$ (Peirce block dimension); scale $= M_Z$.

The factor $d_P/h^\vee(F_4) = 8/9$ also appeared in P108 where $f_V^2 = d_P \times \pi = 8\pi$ set the vector meson decay constant — confirming $d_P = 8$ as a load-bearing constant of the $J_3(\mathbb{O})$ Peirce geometry.

6.2 Summary

1. **EM renormalization (P113 mechanism)** is negligible for all light quarks ($< 0.3\%$).
2. **QCD Peirce block correction** $\delta = (8/9)\alpha_s(M_Z)/\pi = 3.356\%$ closes m_d ($+0.16\%$) and m_s (-0.10%).
3. **u quark:** orbit prediction 2.410 MeV lies within PDG 1σ band; scheme correction brings residual to $+7.83\%$ (0.34σ). First-principles u -quark mass derivation from confinement-scale structure remains open.

6.3 Open items

- **u quark from confinement scale:** derive m_u^{phys} from $J_3(\mathbb{O})$ at $m_{\text{conf}} = \pi \mu_0 m_e$ (P106). The proximity $m_u \sim m_{\text{conf}}$ is the structural hint.
- **Scale selection principle:** derive the choice M_Z (for d,s) vs. m_b (for b) from Peirce sector geometry.
- **OP1-NLO:** b_1 coefficient from Jordan loops (unblocks α_s NLO corrections throughout).
- **SU(3)-breaking:** $f_\omega^2, f_\phi^2 \neq 8\pi$ attributed to strange-quark Peirce weight (P108, open).