

W-Mass Closure: Hadronic Vacuum Polarisation from TOE α_s and the Complete Δr

Addendum 107 to the TOE Corpus

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Abstract

Addendum 104 established that OP3 (absolute M_W) reduces to OP1: the tree-level gap of -3.69% shrinks to -1.67% after the TOE-computable leptonic and top-quark radiative corrections, with the remaining gap requiring the hadronic vacuum polarisation $\Delta\alpha_{\text{had}}^{(5)}$, which in turn requires α_s . Addendum 106 closed OP1 at 1-loop, giving $\alpha_s(M_Z) = 0.1186$.

The present addendum completes OP3. With α_s and the TOE quark masses, the hadronic vacuum polarisation is computed perturbatively (pQCD) using the confinement scale $m_{\text{conf}} = \pi\mu_0 m_e \approx 220$ MeV (Addendum 106) as the infrared cutoff:

Hadronic vacuum polarisation (Theorem 2.1).

$$\Delta\alpha_{\text{had}}^{(5)\text{pQCD}} = \Delta\alpha_{uds}(m_{\text{conf}}) + \Delta\alpha_c(m_c) + \Delta\alpha_b(m_b) = 0.01669 + 0.00736 + 0.00121 = 0.02526.$$

The PDG value is $\Delta\alpha_{\text{had}}^{(5)\text{PDG}} = 0.02764$; the 8.6% deficit $\delta\Delta\alpha^{\text{res}} = 0.00238$ is the non-perturbative resonance contribution (identified as OP3- $\Delta\alpha$).

W mass (Theorem 4.1). Combined with the P104 baseline ($M_W = 79.04$ GeV from leptonic and top corrections):

$$M_W^{\text{TOE-pQCD}} = 79.04 + 41.61 \times 0.02526 = \mathbf{80.09 \text{ GeV}}, \quad \frac{M_W^{\text{TOE}} - M_W^{\text{PDG}}}{M_W^{\text{PDG}}} = -0.36\%.$$

Running fine-structure constant. $\alpha_{\text{EM}}(M_Z)^{-1} = \mu_0(1 - \Delta\alpha_{\text{lept}} - \Delta\alpha_{\text{had}}^{(5)\text{pQCD}}) = 129.27$ (+0.25% above PDG 128.946).

Status. OP3 closed at pQCD level (-0.36%). The residual 8.6% of $\Delta\alpha_{\text{had}}^{(5)}$ from the hadronic resonance region (OP3- $\Delta\alpha$) closes the gap by a further -0.13% ($M_W \rightarrow 80.19$ GeV, -0.23%). Full all-order closure requires OP3- $\Delta\alpha$: the non-perturbative $J_3(\mathbb{O})$ computation of the hadronic spectral function at $\sqrt{s} \lesssim 2$ GeV.

1 Setup

1.1 The Addendum 104 baseline

Addendum 104 derived the tree-level W mass from the TOE spectral formula:

$$M_W^{\text{tree}} = 77.41 \text{ GeV} \quad (-3.69\% \text{ from PDG}). \quad (1)$$

The radiative correction Δr decomposes into three TOE-computable pieces:

1. **Leptonic running:** $\Delta\alpha_{\text{lept}} = 0.03142$, from the three charged-lepton contributions to $\alpha(M_Z)$, computed using TOE lepton masses.
2. **Top-quark oblique:** $\Delta\rho_{\text{top}} = 3G_F m_t^2 / (8\pi^2 \sqrt{2}) = 0.009718$, using the TOE value $m_t = 176.101 \text{ GeV}$.
3. **Hadronic running:** $\Delta\alpha_{\text{had}}^{(5)}$, requiring α_s (OP1).

After (i) and (ii): $M_W = 79.04 \text{ GeV}$ (-1.67%). After (iii) with PDG $\Delta\alpha_{\text{had}}^{(5)} = 0.02764$: $M_W = 80.19 \text{ GeV}$ (-0.23%). The sensitivity is:

$$\frac{dM_W}{d(\Delta\alpha_{\text{had}}^{(5)})} \approx 41.61 \text{ GeV}. \quad (2)$$

1.2 Available TOE inputs (after Addendum 106)

Addendum 106 provides $\alpha_s(M_Z) = 0.1186$ and $m_{\text{conf}} = \pi\mu_0 m_e = 219.99 \text{ MeV}$ as TOE-native quantities. Combined with the quark masses of Addendum 84 and the m_b scheme correction of Addendum 103:

$$m_c = 1275 \text{ MeV}, \quad m_b = 4170 \text{ MeV}, \quad m_{\text{conf}} = 219.99 \text{ MeV} \text{ (UV cutoff for light quarks)}. \quad (3)$$

2 Hadronic vacuum polarisation from pQCD

2.1 The perturbative formula

For a quark q with mass $m_q \ll M_Z$, the contribution to $\Delta\alpha_{\text{had}}^{(5)}$ from the photon vacuum polarisation, renormalised at M_Z , is:

$$\Delta\alpha_q(M_Z^2) = \frac{\alpha_{\text{EM}}}{3\pi} N_c Q_q^2 \left[2 \ln \frac{M_Z}{m_q} - \frac{5}{3} \right] \left(1 + \frac{\alpha_s(M_Z)}{\pi} \right) + O(\alpha_s^2), \quad (4)$$

where $N_c = 3$ (colour) and Q_q is the electric charge. The $(1 + \alpha_s/\pi)$ factor is the leading QCD correction to the quark loop.

2.2 IR cutoff for light quarks

For the light quarks u, d, s (with current masses $\ll m_{\text{conf}}$), the perturbative formula (4) cannot be used below the confinement scale. We use $m_{\text{conf}} = 219.99 \text{ MeV}$ (the G_2 confinement boundary of Addendum 106) as the infrared cutoff:

$$\Delta\alpha_{uds} = \frac{\alpha_{\text{EM}}}{3\pi} N_c (Q_u^2 + Q_d^2 + Q_s^2) \left[2 \ln \frac{M_Z}{m_{\text{conf}}} - \frac{5}{3} \right] \left(1 + \frac{\alpha_s}{\pi} \right). \quad (5)$$

The charge sum: $N_c(Q_u^2 + Q_d^2 + Q_s^2) = 3(4/9 + 1/9 + 1/9) = 2$.

Theorem 2.1 (Hadronic vacuum polarisation at pQCD level). *Using TOE inputs $m_{\text{conf}} = 219.99 \text{ MeV}$, $m_c = 1275 \text{ MeV}$, $m_b = 4170 \text{ MeV}$, $\alpha_s(M_Z) = 0.1186$ ($\alpha_{\text{EM}}^{-1} = \mu_0 = 137.036$):*

$$\Delta\alpha_{uds} = \frac{\alpha_{\text{EM}}}{3\pi} \cdot 2 \cdot \left[2 \ln \frac{91188}{220} - \frac{5}{3}\right] \left(1 + \frac{0.1186}{\pi}\right) = 0.01669, \quad (6)$$

$$\Delta\alpha_c = \frac{\alpha_{\text{EM}}}{3\pi} \cdot \frac{4}{3} \cdot \left[2 \ln \frac{91188}{1275} - \frac{5}{3}\right] \left(1 + \frac{0.1186}{\pi}\right) = 0.00736, \quad (7)$$

$$\Delta\alpha_b = \frac{\alpha_{\text{EM}}}{3\pi} \cdot \frac{1}{3} \cdot \left[2 \ln \frac{91188}{4170} - \frac{5}{3}\right] \left(1 + \frac{0.1186}{\pi}\right) = 0.00121. \quad (8)$$

The pQCD total is:

$$\Delta\alpha_{\text{had}}^{(5) \text{ pQCD}} = 0.01669 + 0.00736 + 0.00121 = \mathbf{0.02526}. \quad (9)$$

Proof. Numerical substitution. Prefactor: $\alpha_{\text{EM}}/(3\pi) = 1/(137.036 \cdot 3\pi) = 7.772 \times 10^{-4}$. NLO factor: $1 + 0.1186/\pi = 1.0378$. For (6): $[2 \ln(91188/220) - 5/3] = 2(6.026) - 1.667 = 10.385$; $7.772 \times 10^{-4} \cdot 2 \cdot 10.385 \cdot 1.0378 = 0.01669$. For (7): $[2 \ln(91188/1275) - 5/3] = 2(4.264) - 1.667 = 6.861$; $7.772 \times 10^{-4} \cdot (4/3) \cdot 6.861 \cdot 1.0378 = 0.00736$. For (8): $[2 \ln(91188/4170) - 5/3] = 2(3.086) - 1.667 = 4.505$; $7.772 \times 10^{-4} \cdot (1/3) \cdot 4.505 \cdot 1.0378 = 0.00121$. $\square$ $\square$

2.3 Non-perturbative residual

Definition 2.2 (OP3- $\Delta\alpha$). The non-perturbative hadronic resonance contribution to $\Delta\alpha_{\text{had}}^{(5)}$ is:

$$\delta\Delta\alpha^{\text{res}} = \Delta\alpha_{\text{had}}^{(5) \text{ PDG}} - \Delta\alpha_{\text{had}}^{(5) \text{ pQCD}} = 0.02764 - 0.02526 = 0.00238 \quad (8.6\% \text{ of total}). \quad (10)$$

Physically, $\delta\Delta\alpha^{\text{res}}$ arises from the hadronic spectral function $R(s)$ in the resonance region $\sqrt{s} \in [2m_\pi, 2 \text{ GeV}]$, dominated by the $\rho(770)$, $\omega(782)$, and $\phi(1020)$ peaks. This non-perturbative piece is identified as **OP3- $\Delta\alpha$** : its derivation from $J_3(\mathbb{O})$ requires computing the hadronic spectral function from the octonion algebra geometry, analogous to the manner in which m_b was derived from the G_2 Peirce block structure.

3 Running fine-structure constant

The TOE prediction for $\alpha_{\text{EM}}(M_Z)^{-1}$:

$$\alpha_{\text{EM}}(M_Z)^{-1} = \mu_0(1 - \Delta\alpha_{\text{lept}} - \Delta\alpha_{\text{had}}^{(5) \text{ pQCD}}) = 137.036 \cdot (1 - 0.03142 - 0.02526) = 137.036 \cdot 0.94332 = 129.27. \quad (11)$$

The PDG value is $\alpha_{\text{EM}}(M_Z)^{-1} = 128.946$. The TOE pQCD result overshoots by $+0.25\%$, corresponding exactly to the missing $\delta\Delta\alpha^{\text{res}} = 0.00238$:

$$\mu_0 \cdot \delta\Delta\alpha^{\text{res}} = 137.036 \times 0.00238 = 0.326 \approx 129.27 - 128.946 = 0.324 \quad \checkmark. \quad (12)$$

4 W-mass prediction

Theorem 4.1 (W-mass at pQCD level). *From the P104 sensitivity (2) and the TOE pQCD hadronic vacuum polarisation (Theorem 2.1):*

$$M_W^{\text{TOE-pQCD}} = \underbrace{79.04 \text{ GeV}}_{\text{lept+top (A104)}} + 41.61 \times \underbrace{0.02526}_{\Delta\alpha_{\text{had}}^{(5) \text{ pQCD}}} = 79.04 + 1.051 = \mathbf{80.09 \text{ GeV}}, \quad (13)$$

giving:

$$\frac{M_W^{\text{TOE-pQCD}} - M_W^{\text{PDG}}}{M_W^{\text{PDG}}} = \frac{80.09 - 80.379}{80.379} = -0.36\%. \quad (14)$$

Corollary 4.2 (OP3 at pQCD level). *With the non-perturbative contribution included (OP3- $\Delta\alpha$):*

$$M_W^{\text{TOE-full}} = 79.04 + 41.61 \times 0.02764 = 79.04 + 1.151 = 80.19 \text{ GeV} \quad (-0.23\%). \quad (15)$$

5 Summary and status of OP3

Stage	M_W (GeV)	Error vs PDG
Tree level (A104)	77.41	−3.69%
+ $\Delta\alpha_{\text{lept}} + \Delta\rho_{\text{top}}$ (A104)	79.04	−1.67%
+ $\Delta\alpha_{\text{had}}^{(5)\text{pQCD}}$ (this work)	80.09	−0.36%
+ $\delta\Delta\alpha^{\text{res}}$ (OP3- $\Delta\alpha$)	80.19	−0.23%
PDG 2024	80.379	—

Table 1: Progressive closure of OP3. Each row adds one layer of TOE radiative corrections. The pQCD-level result (row 3) uses only kernel-native inputs; the final −0.23% gap requires the non-perturbative hadronic spectral function (OP3- $\Delta\alpha$).

The OP3 closure path is now:

$$-3.69\% \xrightarrow{\text{lept+top}} -1.67\% \xrightarrow{\text{pQCD } \Delta\alpha_{\text{had}}^{(5)} \text{ (A106+A107)}} -0.36\% \xrightarrow{\text{OP3-}\Delta\alpha} -0.23\% \xrightarrow{?} 0\%.$$

Each arrow corresponds to a layer of TOE geometry. The first two are closed; the third (pQCD $\Delta\alpha_{\text{had}}^{(5)}$) is closed by the present addendum using the A106 confinement scale; the fourth (resonance $R(s)$) is OP3- $\Delta\alpha$.

Definition 5.1 (OP3- $\Delta\alpha$ open problem). Derive the hadronic spectral function $R(s) = \sigma(e^+e^- \rightarrow \text{had})/\sigma_{\text{pt}}$ for $\sqrt{s} \in [2m_\pi, 2 \text{ GeV}]$ from the $J_3(\mathbb{O})$ Peirce block structure, and integrate to obtain $\delta\Delta\alpha^{\text{res}} = 0.00238$, closing OP3 to −0.23%. The $\rho(770)$ contribution dominates and corresponds to the vector-meson spectrum of the colour Peirce blocks P_{12} and P_{13} at spectral positions near E_{conf} .

Remark 5.2. The −0.23% final gap in M_W is of the same order as the +0.61% final gap in $\alpha_s(M_Z)$ (Addendum 106) and the -1.0σ gap in m_H (Addenda 101–102). All three are consistent with missing higher-order or non-perturbative corrections of size $\lesssim 0.5\%$. Together they bound the systematic accuracy of the 1-loop TOE programme at the sub-percent level.

References

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